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THE ASTROPHYSICAL JOURNAL

AN INTERNATIONAL REVIEW OF SPECTROSCOPY AND ASTRONOMICAL PHYSICS

Founded in 1895 by GEORGE E. HALE *and* JAMES E. KEELER

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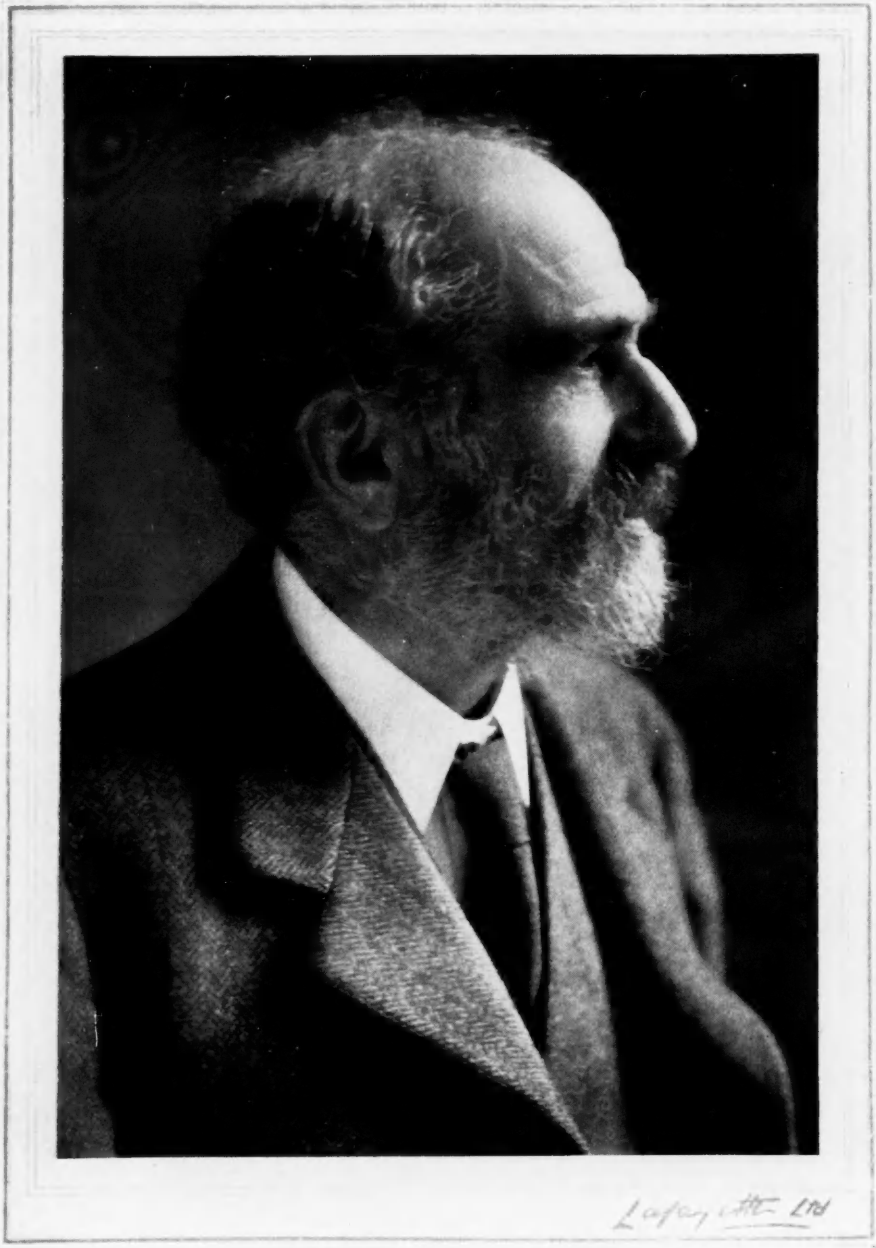

SIR ARTHUR SCHUSTER

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SIR ARTHUR SCHUSTER

By GEORGE E. HALE

Since the establishment of the *Astrophysical Journal* forty years ago, we have lost by death many members of our international board of editors. Keeler, Cornu, Dunér, Hastings, Huggins, Michelson, Pickering, Rowland, Tacchini, Vogel, Young, Nichols, Runge, Belopolsky, Pérot, Riccò, Schwarzschild—the list of our former collaborators is long and honorable, impossible to review without emotion by one who knew them all from the beginning. And now we mourn the loss of still another friend and collaborator, Sir Arthur Schuster, who died on October 14, 1934, at his country house in Berkshire.

Arthur Schuster was born at Frankfort on September 12, 1851, and his early education was at the Frankfort Gymnasium and the Geneva "Academy." His first boyhood inclination was for architecture, but at the Academy he was led toward the physical sciences by Plantamour in astronomy and by Soret in molecular physics. In 1870 he left Geneva for Manchester, where his life work in physics was begun.

The origin of his epoch-making career in Manchester, where his father had large interests in the cotton industry, is best described in the words of Sir Henry Roscoe and in those of Schuster himself, in the pamphlet recording the *Commemoration of the 25th Anniversary of the election of Arthur Schuster, F.R.S., to a Professorship in the*

Owens College.[1] Sir Henry, who presided at this meeting, opened his address as follows:

It was in the autumn of 1870, therefore thirty-six years ago, that I first saw Arthur Schuster. He was then nineteen years of age, and had entered my evening class in Chemistry. About this time his father called upon me to ask my advice as to his son's future. It seems that the young man wished to follow a scientific career, and to his doing so, his father not unnaturally demurred, unless he could be assured not only that Arthur was in earnest, but that he possessed the qualities which would make for success. My reply was "Why not? Your son has evidently made up his mind, and as for his capabilities, you need have no fear on that score." So Arthur Schuster became a physicist, and one of whom all Science is proud. I take no credit for myself for having implanted the love of nature in his heart—this was done by Marignac, Soret, and Plantamour in Geneva. All I did for him was perhaps to stimulate his action, and to obtain for him his father's permission to leave the easy path of pecuniary gain which lay open to him, and to take the uphill road which alone leads to eminence in Science.

Professor Schuster, in responding, replied:

Those who have read your Memoirs which have lately appeared will remember a chapter entitled "Bunseniana." If ever I should be induced to publish a similar sketch of the scientific events of my life, I think the title of one of its principal chapters would have to be "Roscoeiana." You referred to the date at which you first made my acquaintance, but I knew you before you knew me, and you are connected with the two most important dates of my career. I made your first acquaintance on the day which I still consider to have been the happiest of my life: the day on which I left school. I had been brought up on the lines of a severely classical education, and when I had reached the limits of knowledge which this so-called "humanitarian" teaching was capable of instilling into me, I was allowed to apply myself for a few months to the studies for which I showed some aptitude. On the day of my deliverance, I was presented by a private tutor with a book which I always look upon with feelings of reverence, and which I have brought with me to-day. It is the German translation of your book on Chemistry, and on its title page bears inscribed the date of April 3, 1868. I still remember the pleasure I felt on opening it when my eye fell on the coloured plate representing the spectra of metals. I think it is to this first impression that I owe my affection for the study of spectrum analysis.

It was on Roscoe's advice that Schuster decided to study at Heidelberg, where he took his Ph.D. under Kirchhoff in 1873. Schuster had a keen sense of humor, and he has told me many diverting stories

[1] The title of the Commemoration Volume is *The Physical Laboratories of the University of Manchester, A Record of 25 Years' Work*. Manchester: University Press, 1906.

of this period. Several of these stories relate to Bunsen, whose remarkable absent-mindedness and other attractive peculiarities were famous among his colleagues and students. It is a misfortune that ill health prevented Schuster from completing his personal reminiscences, which covered so wide a span.[2]

Once more in Manchester, Schuster became Balfour Stewart's assistant, afterward returning to Germany for further study at Göttingen and Berlin. His interest in astrophysics was now fully aroused and among his many activities he led a very successful eclipse expedition to Siam. Subsequent eclipse expeditions in which he took part were in Colorado (1878), Egypt (1882), and the West Indies (1886).

As the successor of Balfour Stewart he made the Physical Laboratory in the Owens College, Manchester, one of the most productive of its period. He had worked for several years at the Cavendish Laboratory, Cambridge, under Maxwell and Lord Rayleigh; and among his own students he could count such men as J. J. Thomson, Eddington, and Rutherford. It was Rutherford who became his successor at Manchester when Schuster retired from the Langworthy professorship of physics in 1907.

Schuster's first paper was on the spectrum of nitrogen, and he never lost his active interest in all phases of spectroscopy. His eminence in this field led me to visit him in Manchester especially to discuss with him the remarkable qualities of the H and K lines of calcium, which he had photographed at the Siam eclipse. I had then hardly begun astrophysical research, and had no reason to expect the kind reception he gave me. This led to a lifelong friendship, and to intimate association in many phases of research, especially in the promotion of international co-operation.

As Schuster's fundamental contributions to knowledge will be widely reviewed by other writers, I can perhaps give this paper its chief value by dwelling on his outstanding work in the international field, where I was most closely associated with him. From his boyhood his interests were so broad that this aspect of his career was a natural one. In commenting on it at the Jubilee Celebration to which I have previously referred, Dr. Griffiths remarked:

[2] Commenced in his *Biographical Fragments*, published by Macmillan in 1932.

> Perhaps I may venture to say that the greatest characteristic of Dr. Schuster is his cosmopolitanism—cosmopolitanism of the highest kind. He has taken a large part in the various International Conferences; and I believe that if it were possible to hold an Interplanetary Conference—meeting, say, on the planet Jupiter—it is very probable that Dr. Schuster would be asked to act as the earth's representative.

Schuster made brief references to co-operative work on the same occasion, but the reader should also refer to his address before the Royal Institution on May 18, 1906, entitled "International Science."[3]

In this paper he points out:

> We may distinguish between three types of international organizations. The first aims simply at collecting information, the second is intended to fix fundamental units or to initiate agreements on matters in which uniformity is desirable, while in the third type of organization a more direct advance of knowledge is aimed at, and research is carried out according to a combined scheme. Generally an international association does not entirely fall within any single one of these divisions, but it is useful to draw the distinction and classify the associations according to the main object which they are intended to serve.

He then describes the more important international organizations of various types, mentioning the International Catalogue of Scientific Literature; the International Chart of the Heavens; the International Bureau of Weights and Measures; the conferences and conventions held for the purpose of adopting a uniform system of scientific units and nomenclature; the task of determining the earth's radius or circumference, which depends largely upon the measurement of arcs of meridian; the study of the acceleration of gravity at many points on the earth's surface; the co-operative measurement of the variation of latitude; the importance of international co-operation in meteorological, seismic, and vulcanological observations; the work of the International Union for Co-operation in Solar Research; and the apparent possibilities of the former International Association of Academies.

There are certain men of science who are by nature so obviously fitted to devote themselves to their individual researches that they cannot be expected to take part in co-operative enterprises. It would be a mistake to divert such men from the subjects of their choice,

[3] Published in the *University Review*, June, 1906.

and the advocates of international co-operation should not attempt to do so. On the other hand, there are men like Schuster who greatly appreciate the possibilities of co-operation, and are willing to devote a considerable share of time to it, in spite of the fact that their first choice would undoubtedly be personal research. Unfortunately a widespread fallacy has prevented many able men from joining in international undertakings of the greatest importance.

This fallacy assumes that co-operation means nothing but machine-like drudgery, even to the extent of sacrificing individual liberty and originality to the interests of some national or international formula. I have no hesitation in saying that co-operative enterprises demanding such sacrifices should not be encouraged, except in cases where the voluntary participants personally prefer to work in this way. Freedom of action and the encouragement of original thought obviously lie at the very base of science, and we cannot afford to dry up the springs of vitality at their source. A Galileo or a Newton or an Einstein cannot be produced by an international conference, nor can lesser men who have nevertheless contributed enormously to original thought. How, then, are we to reconcile our co-operative projects with the prime necessity for personal freedom?

Some of these projects avowedly demand a vast amount of routine work, carried out in accordance with a systematic and unvarying plan. Such a case we find, for example, in the International Chart of the Heavens. As projected in 1887, eighteen observatories, widely distributed throughout the northern and southern hemispheres, volunteered to take part in this work. Each was assigned a certain region of the sky, which was to be photographed repeatedly with telescopes of the same type, aperture, and focal length. The measurements of the positions of the star images were also to be conducted on a common plan.

Stellar photography was still in an early stage at that date, and many observatories were planning to determine in practically the same way the positions of stars in large areas of the heavens. With such a great amount of routine work inevitable, it was a happy idea on the part of Sir David Gill and the astronomers of the Paris Observatory to unify the task so as to depict the entire sky and to avoid unnecessary duplication. Although the results were to be reduced and

published on a uniform basis, there was nothing to prevent any observatory from following up and publishing separately discoveries made on its own plates, such as proper motions, variable stars, etc. Moreover, the necessary work of measurement and reduction, though long and tedious, is inevitable in many kinds of research. Assuming, finally, that a wise distribution of the task was made among observatories and astronomers best qualified to deal with it, and recognizing that their co-operation was purely voluntary, we can appreciate why this immense scheme was undertaken. It has been in large part completed, though subsequently improved instruments and methods led several observatories to initiate other important projects of star-charting by photography. This was obviously essential in any event, as the faintest objects to be measured (those on the short-exposure photographs) included only the brighter stars down to the 11th magnitude.

Thus, even in this extreme case, most of the international co-operative projects involve no more injury to personal initiative and original thought than is inherent in the nature of the work done at many observatories. The chief problem is to shape each co-operative undertaking in such a way as to inspire every participant to make his best contribution to science, in whatever field or in whatever manner he may work. In many an international project, tasks that look like the dullest routine may, and often do, result in great discoveries.

One of the most important needs of science is to establish closer relationships between workers in different fields. It is comparatively easy to bring together specialists in given subjects and to secure their friendly co-operation. But to fill the gaps between the various branches of science is a more difficult task, in spite of the obvious possibilities of advance. Such possibilities are shown by the development of astrophysics, geophysics, biochemistry, and many other subjects. However, the fact remains that countless opportunities are lost because instruments, methods, and ideas which have originated in some particular field are unknown or at least unused in other fields.

Schuster was deeply impressed by such considerations, and therefore welcomed the formation in 1898 of the International Association of Academies, in which he took an active part. Many of my associa-

tions with him go back to the meetings of the Council and the triennial assemblies of this International Association, the last one of which occurred at St. Petersburg in 1913. I recall this meeting with special clarity for several reasons, the chief of which related to the outbreak of the World War in the following year. Like many other institutions, the International Association of Academies did not survive the war; but a joint study of its procedure on many occasions made clear the major obstacles which so greatly hampered its possibilities.

Schuster and I had had other opportunities to study the problems of international science, especially in connection with the International Union for Co-operation in Solar Research, which was initiated by a committee appointed in 1904 by our National Academy of Sciences. After much correspondence with leading astronomers and physicists in this country and abroad, a preliminary meeting was held in connection with the International Congress of Science at the St. Louis Exposition. Representatives of fourteen widely distributed academies and societies were present, and the desirability of establishing a permanent International Union was unanimously voted. An executive committee, with Schuster as chairman, was appointed to organize a second conference, which was held at Oxford in September, 1905. The International Union for Co-operation in Solar Research was then formally established, and Schuster was made chairman of its small Executive Committee.

Although scientific papers, as well as the reports of its various committees, were often read at the triennial meetings, the chief purpose of the Solar Union was to organize active co-operation in research, without interfering with personal liberty. Thus the participants in all of its projects, which included the determination of standard wave-lengths, were encouraged to devise and utilize all instruments and methods that seemed to them most promising.

If space permitted, it would be interesting to outline the many advances which have resulted from this policy. The four volumes of *Transactions* published under Schuster's editorship tell the story. As I review them, I recall many meetings with Schuster at the Athenaeum in London, in Manchester, and at his country house near Twyford, and also at Oxford, Cambridge, Paris, Rome, Vienna, St. Petersburg, Lake Garda, and Venice. He greatly enjoyed painting, and

usually carried his complete outfit with him. In this connection I especially remember him standing before his easel in his room on the Grand Canal in Venice, which commanded a superb view of San Giorgio and the sparkling waters lapping the walls of the Piazzetta.[4]

During all this period prior to the war the wider possibilities of international co-operation in research were becoming clearer. Throughout the war Schuster, who had become a naturalized British subject soon after his establishment in Manchester, was always completely loyal to the country of his adoption. His judgment of the events of 1914–18 was that of the calm and well-balanced Englishman, and his son naturally served in France with the British Army, while he and the other members of his family did their full part in England.

Meanwhile, as secretary of the Royal Society, he continued his correspondence with the representatives of several academies on international co-operation in research. He recognized that the bitterness of feeling between the Allied and Central Powers could be overcome only by the lapse of time, and that too early an attempt to renew scientific relations with Germany and her associates would inevitably result in intensifying the antagonism. The Royal Society and the Paris Academy of Sciences called an inter-Allied conference in London in 1918, where there was a general discussion of the problem, followed by a further debate in Paris a few weeks later. During these conferences Schuster played a vital part, and his long experience was invaluable.

The situation which we faced may be easily illustrated. In astronomy there had been many international committees, dealing independently with the international chart of the heavens, solar research, Kapteyn's Selected Areas, time standards, astronomical ephemerides, the distribution of astronomical telegrams, the orbits of minor planets, and other subjects. Had not the time arrived to initiate what might ultimately become an International Astronomical Union, bringing all of these groups into a single organization? The International Union for Co-operation in Solar Research, which since

[4] For a set of reproductions in color of some of Schuster's paintings, see his *Indian Sketches*, privately printed in 1908.

1905 had grown to include almost all phases of astrophysics, had proved that such united efforts were both desirable and productive. In geophysics distinct organizations dealt with geodesy, meteorology, terrestrial magnetism, seismology, and other branches of a subject of wide scope, and there was need of a joint organization capable of securing common consideration of major problems embracing several aspects of this science. Similar needs existed in other fields. Finally, to arrange for effective organization and active co-operation in studies of still greater range, a central body somewhat similar to the former International Association of Academies was required. This must be constituted, however, in such a way as to secure a much wider participation of the best-qualified men of science than was feasible under the old plan.

The outcome of these conferences was the establishment of the International Research Council, with its constituent Unions of Astronomy, Geodesy and Geophysics, Chemistry, Mathematics, Physics, Scientific Radio, Geography, and the Biological Sciences. Under the presidency of M. Picard, with Schuster as general secretary, the International Research Council initiated the various Unions, most of which have been very successful. After failing health forced Schuster to retire from the position of general secretary, his place was taken by Sir Henry Lyons. More recently, the central organization changed its name to the International Council of Scientific Unions, and each Union was represented on its executive committee by two members. Subsequent details are unnecessary here, as they do not fall within the period of Schuster's active participation.

In addition to his extensive international work, Schuster's broad interests and exceptional knowledge enabled him to advance many branches of science in England. As a leading member of committees on seismology, meteorology, and other branches of geophysics, and especially as secretary of the Royal Society, he exerted a wide influence of lasting value.

With so much to his credit as an organizer and administrator, a reader unfamiliar with Schuster's personal researches might be left with a one-sided view of his eminence in science. Space fails me to review these researches in detail, but some of them should be men-

tioned, especially those described in the address of the president of the Royal Society as the reason for the award of its highest honor (the Copley Medal) to Schuster in 1931.

As Sir Gowland Hopkins then pointed out, we owe to Schuster the theory of the method of determining the ratio of charge to mass of cathode rays and an approximate experimental measurement of this ratio. This fundamental investigation was accompanied by many other equally important researches on the electric discharge in gases. In astrophysics Schuster's contributions were vital, including, as they did, theoretical researches on the law of distribution of spectral lines, the resolving power of spectroscopes, the periodicities of sun-spots, the relationship between terrestrial magnetic storms and solar phenomena, the nature of the solar atmosphere, etc. He also made extensive investigations of terrestrial magnetism and other branches of geophysics. His invention of the periodogram method of determining periodicities in statistical material has been applied, not only to his own studies of sun-spots and other physical phenomena, but by investigators in many other fields of science and also in economics.

Naturally Schuster was the recipient of many honors from leading institutions at home and abroad. His numerous friends will miss him keenly, especially those who are in a position to recognize how many rare and inspiring qualities he combined.[5]

CARNEGIE INSTITUTION OF WASHINGTON
MOUNT WILSON OBSERVATORY
November 1934

[5] For a valuable account of Schuster's life, containing many important details not included here, see an article in *Nature* for October 20, 1934.

THE CLASSIFICATION OF STELLAR SPECTRA

By H. N. RUSSELL, CECILIA H. PAYNE GAPOSCHKIN, and D. H. MENZEL

ABSTRACT

The criteria employed in the existing Draper classification of spectra are detailed; the problems of more general criteria and of specific peculiarities are considered; Struve's recent suggestions are canvassed; and the physical prerequisites, and taxonomic principles, upon which a definitive classification should depend, are discussed. It is concluded that such a classification should be deferred for the present.

I. INTRODUCTION

The problem of classifying stellar spectra has been brought into new prominence by Struve's recent discussion,[1] and by his outline of the problems recently undertaken at the Yerkes Observatory.[2] As open discussion of the subject seems relevant at this time, the present paper considers some of the points on which Struve has touched, and attempts to outline the general principles of classification for stellar spectra.

In a study made eight years ago by one of the writers, the following summary of the principles of classification was given:

> In classifying a number of objects, an attempt should be made to select criteria that will distribute the material in the most natural groups. A classification devised from one point of view will not necessarily appear natural from another, and the best that can generally be done is to select the standpoint that seems to be the most important. From all other standpoints the classification is empirical, and must be treated as such. . . . .
>
> The descriptions that are contained in the preface to the *Henry Draper Catalogue*, and which have long been classical, were designed to describe the salient features of the groups that had been formed. It is only in a somewhat restricted sense that they constitute the criteria for those groups. The descriptions were compiled from the spectra of apparently bright stars . . . . , but the greater number of the spectra actually classified are taken with short dispersion. . . . .[3]

The classification of stars is very largely a *practical* problem.

Instead, then, of examining the possible merits of the best theoretical classification system, it appears to be more useful to examine the physical implica-

[1] *Ap. J.*, **78**, 73, 1933.

[2] *Pop. Astr.*, **41**, 543, 1933.

[3] Payne, *Stellar Atmospheres*, 1925.

tion of the most representative classification that it has been found possible to make in practice.

II. THE DRAPER CLASSIFICATION

The Harvard system is the product of the experience of a group, headed by Pickering and Miss Cannon,[4] who have looked at a greater number of different stellar spectra than any other group; from this standpoint alone it must be recognized as having a maximum representativeness. But from its first days this system served only to place the spectra in convenient pigeonholes, from which those worthy of special study could be withdrawn, and redistributed with labels, such as Miss Maury's "a," "b", and "c," and the numbers referring to the voluminous remarks to Volumes **28** and **56** of the *Harvard Annals*. As far as the fainter stars are concerned, the original Draper classification, augmented where possible by the later suggested[5] prefixes "g" and "d," will probably represent the spectra with adequate accuracy, until the photographic process has been greatly expedited, except for some of the rare classes such as O and W and perhaps M and N. The question of revision or replacement of the Draper system concerns only the brighter stars, where high dispersion brings out important additional spectral detail.

When the Draper system is considered from the astrophysical point of view, the criteria are seen to be multiple. The M, N, R, and S stars are classified chiefly by the strengths of the bands; the atomic lines are badly blended and difficult to analyze. The criteria are the most indefinite for the K stars; for the fainter objects of this group the intensity to the violet of the G band is used, so that the classification is based upon energy distribution. For the G stars the criterion is chiefly one of temperature, depending mainly on the relative intensities of the iron and hydrogen lines. In the F and, particularly, the A classes, the criteria are usually founded on the relative intensities of the lines of hydrogen and ionized calcium. Here the dependence is upon ionization, complicated by opacity effects, and is especially sensitive to the relative abundance of hydrogen. The later B stars are usually classified by the relative intensities of 4471 *He* I and 4481 *Mg* II, for which temperature and ionization are the controlling factors; this criterion is giant and dwarf sensitive. For the early B

[4] Cf. Preface, *Harvard Ann.*, **91**, 1918.

[5] *Trans. Int. Astr. Union*, **2**, 1925.

stars and the O stars, high-stage ionization is employed. The old criteria for the Wolf-Rayet stars must be replaced by others depending upon excitation and composition.

Multifarious as these criteria are, they express the most *conspicuous* features from type to type. It is doubtful whether more outstanding bases for classification could be selected. This is particularly true when low-dispersion plates are employed, as in most of the work contained in the *Henry Draper Catalogue*.

Conflict between the various criteria is to be expected in certain cases. In some groups—notably the "c" stars—additional spectral characteristics, such as the width and strength of the hydrogen lines, are used.

III. THE GENERAL PROBLEM

There are many problems associated with the appearance of stellar spectra that still remain to be solved. The general features, such as the variation of line intensity along the spectral sequence, and the absolute-magnitude effects, can be accounted for by a fairly simple theory, but the problem of line profiles, central intensities, and strengths of individual lines has scarcely been touched. In view of the wide variation in gradient of lines from the same multiplet, as observed by Struve in different stars,[6] it is evident that the elementary theory demands extensive modification. The difficulties may arise from uncertainties in the atomic coefficient of line absorption or in the approximations introduced into the solutions of the equations of radiative transfer, from physical complications such as turbulence, and perhaps from other causes. Until these outstanding problems are solved, spectral classification must continue to proceed on bases that are largely empirical. It should be emphasized, furthermore, that if ever we develop a theory sufficiently exact and elastic to account for all of the features of stellar spectra, classification will be either unnecessary or will evolve into a shorthand listing of the parameters necessary to define the exact solution of the problem.

Many of these parameters can already be recognized:

a) The most important are the flux of energy per unit area (which determines the effective temperature) and the surface gravity. There

[6] *Ap. J.*, **74**, 225, 1931.

is good theoretical reason for concluding that two stars with the same values of these parameters and with atmospheres of the same composition should have identical spectra except for the subordinate influences, such as rotation, discussed below (§ *c*). The effective temperature is undoubtedly the primary parameter in the Draper sequence, while differences in gravity determine the giant and dwarf distinction. The possibility of determining absolute magnitudes from the latter peculiarities depends on the mass-luminosity correlation, so that it seems preferable to refer to them as effects of gravity rather than of absolute magnitude. Gravity seems also much better as a parameter than "pressure," for the latter varies enormously within the atmosphere and its effective mean value depends upon the general opacity and so upon the composition. Phenomena due to the Stark effect, such as the widening of the hydrogen lines, and the presence of forbidden helium lines detected by Struve,[7] belong primarily under this head.

These parameters could theoretically be precisely stated by two numbers, and it might be possible to develop a strictly spectroscopic classification in which their values were clearly exhibited, the temperature being found from the relative intensities of lines of different excitation potential,[8] and the effective pressure from the level of ionization; but such spectra as that of Antares show that grave theoretical difficulties have still to be met.

These parameters may be objected to on the grounds that they are not directly observable and can generally be deduced only by application of theory. It seems necessary, however, in this case to resort to theoretical considerations to separate the effects of these two independent variables, which otherwise would be hopelessly entangled.

b) The next major set of differences, depending on the atomic composition of the material, are unfortunately incapable of any such simple numerical statement. In certain specific cases the conspicuously observable features of the spectrum depend primarily upon a single abundance ratio. The most conspicuous example is the branching of the sequence into the "oxygen stars" of class M and the "car-

[7] *Ibid.* [8] Adams and Russell, *Mt. W. Contr.*, No. 359; *Ap. J.*, **68**, 9, 1928.

bon stars" of classes R and N, where the distinction depends on the relative abundance of the constituents of the tightly bound molecule, carbon monoxide.

Another important example is found in classes F5 to A0, where the relative abundance of the easily ionized metals and the difficultly ionized permanent gases may produce great differences in the "depth" of the photosphere and in the relative strength of various lines, and possibly account even for "c" stars like α Cygni.[9] Here, again, the wide gap between the ionization potentials of the two groups and the enormous preponderance of hydrogen cause this particular difference in composition to be spectroscopically predominant in the given range.

The third great distinction, between the "carbon" and "nitrogen" series of Wolf-Rayet stars, is not yet interpreted theoretically.

There remain many other "peculiarities," such as the unusual strengh of lines of Si^+ in some stars, of Sr^+ in others, and of Ba^+, Mn^+, or Eu^+ in a few, which are probably explicable by actual differences in abundance. The strength of *ZrO* in the S stars is probably a similar phenomenon, and there may be other instances in classes R and N.

The very nature of these differences makes a numerical classification inappropriate. The distinction between the "carbon" and "oxygen" stars is likely, theoretically, to be pretty sharp,[10] and the existing classification of M, R, and N stars appears adequate. A good deal more work both in theory and observation will be required before the hydrogen abundance can be treated adequately. The minor cases should probably remain as peculiarities. The notation already suggested for them[11] seems about as simple as is practicable.

c) A number of unrelated modifying factors remain to be considered.

Rotation introduces a distinctive type of widening which can often be definitely recognized (as in α Aquilae).[12] It might be objected

[9] Russell, *Mt. W. Contr.*, No. 477, p. 59; *Ap. J.*, **78**, 297, 1933.

[10] Russell, *Mt. W. Contr.*, No. 490; *Ap. J.*, **79**, 340, 1934.

[11] *Trans. Int. Astr. Union*, **3**, 167, 1928.

[12] Shapley and Nicholson, *Mt. W. Comm.*, No. 63, 1919; Shajn and Struve, M.N., **89**, 222, 1929; Carroll, *ibid.*, **93**, 680, 1930.

that the spectrum of a rapidly rotating star would be classed differently if viewed from the equator and from the pole. The answer is that the spectra really are different.

Turbulence, as Struve and his collaborators have recently suggested,[13] may have an important influence in altering the gradient of intensity between the stronger and weaker lines. If this effect should prove to be closely correlated with absolute magnitude or surface gravity, it may ultimately be incorporated under (*a*).

The actual *size* of a star may be important if the depth of the atmosphere is a sensible fraction of the radius, so that gravity varies from point to point. This may be the case in Wolf-Rayet stars and some other bright-line stars,[14] and also in some normal stars where the chromosphere is unusually extensive.

Interstellar absorption is undoubtedly responsible for detached lines of Na and Ca^+. If the interstellar medium were uniform, these would afford excellent criteria of distance; but there is strong evidence that it is patchy, so that spectroscopically they must be regarded as involving an independent parameter.

Color excess belongs really to the realm of spectral photometry and not to the classification of line spectra. It is, however, so intimately connected with the latter that it deserves mention. It is probably of complex origin, arising partly from space absorption and partly from conditions within the star. For purposes of classification it can be adequately handled by the specification of a color index—or, eventually, more than one—for different wave-lengths. The line spectrum should be independently classified. There is some evidence pointing toward an excess of radiation in the extreme ultra-violet, at wave-lengths shorter than about λ 1000 A, over that to be expected from the star's effective temperature as judged from direct spectrophotometry. In some cases this radiation may be sufficiently intense to affect the spectrum appreciably.

IV. STRUVE'S SUGGESTIONS

In several papers, notably the one dealing with the principles of classification, Struve has rearranged the B stars in "giant" and

[13] Struve and Morgan, *Proc. Nat. Acad.*, **18**, 585, 1932; Struve and Elvey, *Ap. J.*, **79**, 409, 1934.

[14] Kosirev, *M.N.*, **94**, 430, 1934; Chandrasekhar, *ibid.*, p. 444.

"dwarf" groups on spectroscopic criteria. His three criteria for a class B dwarf differ in kind. The first (line wings) distinguishes the "normal" B from the cB star, or supergiant. The "forbidden" helium lines demand further investigation to determine why some appear more strongly than others, but the correlation of the phenomenon with absolute magnitude is probably well enough established to warrant its use as an index of luminosity. The third criterion (interstellar calcium) is admittedly statistical in character, and while it may therefore assist us in selecting groups of stars that are on the average brighter or fainter than other groups selected with the same lines as criterion, it cannot be safely used to determine the brightness of an individual star. The work of Beals[15] on the relative intensities of interstellar calcium and sodium lines indicates the uncertainties involved if the intensity of interstellar calcium is used in determining individual luminosities.

It would be expected that Struve's individual criteria would distinguish between groups of stars that differ systematically in brightness, and this is indeed the case. His "giant" and "dwarf" groups are in fact the well-known groups of supergiant and normal B stars. The "c" character is no longer a discriminant for stars of class B1 and earlier, possibly because of the prevalence of high rotational speeds for stars of these types; but for classes B2 and later, *all* the stars enumerated by Struve are recognized supergiants (9 Cep*, cB9; 55 Cyg*, cB2; o^2CMa, cB5; 67 Oph, cB5; β Ori, cB8; α Cyg, cA0; σ Cyg, cA2). The two stars with asterisks are in addition well-known "low-temperature" stars, to which group ζ Persei, also given in Struve's "giant" group and probably a supergiant, also belongs. Remarks on the peculiarities of many stars, including those enumerated above, are contained in the little-read remarks to *Harvard Annals*, Volumes **28** and **56**.

That Struve's classifications, while producing groups that differ systematically in brightness, are not adapted for individual determinations of luminosity is shown by examining his table of "dwarfs." It is difficult, from their spectra and temperatures, to regard 4 Lacertae, τ^9 Eridani, η Leonis, and 13 Monocerotis as anything but supergiants; and Struve himself admits in a footnote that three of these

[15] *Ibid.*, **93**, 585, 1933.

stars are generally so classed. The result is not unexpected, and indicates that the criteria used are adapted only to the formation of groups, not to the study of individuals. The wings of the hydrogen lines might form a good individual criterion, if it were certain that there were no rotation effects for any of the stars examined. In the absence of this certainty, the criterion becomes of little use in practice.

Struve's discovery, previously referred to, that lines of the same multiplet may show different intensity gradients in different stars, is very important. The lines of *O* II and *He* I, in early-type spectra, show the effect markedly. Struve's original suggestion, that the intensification is correlated with series multiplicity, is not borne out by further study of the lines of *O* II, and it seems probable that the same is true for *He* I. In any event it is certain that the effect is overemphasized by the fact that Struve compared singlets and triplets of neighboring wave-lengths rather than series members with the same quantum numbers. Thus 4388 should be paired with 4026, and 4922 with 4471. The gradation of multiplet intensity may prove to be a valuable adjunct to classification, but, in the absence of a satisfactory theory, it can be so employed only if its correlations are determined.

Struve asks: "If two stars of equal atmospheric temperature and pressure are given, are their spectra necessarily identical?" The answer, of course, is "No," unless the surface gravities, relative atomic abundances, and other factors are all identical, which we have every reason to believe is seldom the case. A complete description of the spectrum of a star requires not only the wave-lengths and intensities of all the lines, but also their profiles; and a tabulation of such data, even for the brighter stars, would be of great value in selecting the necessary basis for a new classification. But we cannot admit that a mere listing of line intensities *constitutes* sufficient classification; classified material differs from tabulated data as a rogues' gallery differs from an undiscriminating photographic album.

V. PREREQUISITES FOR A DEFINITIVE CLASSIFICATION

Further classification of the spectra of bright stars is without question desirable, but caution must be exercised against premature

extension or replacement of the present system, which with all its limitations is highly satisfactory. There are certain necessary preliminaries to a new classification of stellar spectra, of which some of the more important are the following:

1. A systematic survey of spectra from λ 3000 to λ 9000. Many important lines of neutral atoms lie in the infra-red, and most of the strong enhanced lines are in the ultra-violet.

2. A precise classification must involve knowledge obtained with large dispersion. The number of stars that can be observed with the coudé arrangement of a large reflector is necessarily limited, but all such information as is available must be utilized in extending the classification to the large numbers of stars not so accessible.

3. A special survey for, and study of, peculiar stars, with particular reference to their frequency and correlations.

4. A study of the spectral distribution of radiation, and of deviations from black-body radiation, for a number of normal and peculiar stars.

5. A theoretical discussion of line formation, line profile, and central intensity. Further study of the chromosphere will undoubtedly throw light on these problems.

6. Accurate measures of intensities, equivalent widths, and line profiles.

7. Extension of the work on abundance equilibrium in stellar interiors, to include, if possible, predictions of the probable composition of the surface layers.

8. Further observations and analysis of the spectra of the sun and sun-spots, with special reference to variations over the solar disk.

9. Photometric analysis of spectroheliograms, which would provide data on the effect of surface features (faculae, flocculi, spots) upon the spectral lines.

10. Study of band spectra in late-type stars, with special attention to the relation between intensity of band absorption and the number of molecules active.

11. A laboratory investigation, supplemented by theoretical calculations, of intensities of individual lines and multiplets, and also of relative intensities of different series members.

12. Laboratory and theoretical studies of collision excitation and

de-excitation, with special reference to their effect on the atomic absorption coefficient.

13. A quantitative survey of the changes in the spectra of individual variable stars.

VI. TAXONOMIC PRINCIPLES

Certain general taxonomic principles long applied in the biological field are relevant in the formation of a classification:

1. A previously established genus or species may be divided, the original name being preserved for a typical part. A valid species may also be divided into subspecies.

2. A name, however, may not be used to connote something *different* from at least a part of what was included in its old signification.

3. Synonyms, whether arising in this or in other ways, are *nomina nuda*, and may not be used in the future to describe any species.

4. Type specimens must be of good quality and capable of exact and detailed description.

The first of these principles has already been used in spectroscopy, e.g., where classes R and S were split off among the red stars. The second is already abundantly illustrated by such refinements as the letters "g," "d," "n," and "s." The third suggests that older symbols such as "Ma," "Oc," etc., should be abandoned altogether (without prejudice, of course, to their use in older books and compilations). The fourth point is of considerable importance. A definitive classification, or even the next step toward it, should certainly be based upon the best available material. Spectra of high dispersion and covering the maximum practicable range of wave-length must be of primary importance in establishing any new criteria or nomenclature. The application of the new or improved classification to spectra of smaller dispersion and resolving power is a subsidiary problem. Less precise classification in such a case is to be expected and need cause no disquiet. A similar usage already prevails in the natural sciences, where "*Carex* sp." may denote the presence of plants belonging to this genus but not specifically determinable with the specimens at hand.

VII. SUMMARY

The desirable features of a system of classification are as follows: It must be based exclusively upon observable characteristics of the *spectra*. Characteristics clearly recognizable in a large number of spectra, but not yet or not fully interpreted physically, are more valuable criteria than difficultly observable details which are supposed to be well understood. The intricacy of the theory of stellar spectra and its rapid development emphasize the need of caution in tying a nomenclature, which is intended to be permanent, to particular theoretical details. For example, in the hotter stars, most of the lines within the necessarily limited spectral range available arise from high excitation levels, in which the atomic concentration may be markedly affected even by minor deviations from thermodynamic equilibrium. Good names, however, such as "oxygen" and "atom," may well survive the theories on which they were based.

The system should not be confined to a limited number of specific parameters. A label of some sort should be available for exceptional objects such as υ Sagittarii, R Coronae Borealis, and η Carinae, as well as for standard objects like K dwarfs.

It is probably a counsel of perfection to expect to find a specific label for every idiosyncrasy, and a fairly liberal use of the letter "p," or some equivalent, is desirable to denote characteristics which are common to very few objects. Some statement of the nature of the peculiarity should always appear in the footnotes to any general catalogue.

Finally, the mere existence of the Draper classification, applied to hundreds of thousands of stars, is a strong argument for its substantial conservation. The fact that the principal parameters of this classification are found to be closely, though not rigorously, related to fundamental properties of the stellar atmospheres themselves makes the argument conclusive.

Certain other characteristics of the stars, such as the distribution of energy in the continuous background, are closely related to their line or band spectra. It seems highly undesirable, however, to use these as criteria in spectral classification, since there is evidence that

the correlation between these quantities and the characteristics of the line spectra is by no means perfect. For example, the abnormal redness of certain B stars arises apparently in the stars themselves; and of others, in space absorption outside them. By deliberately keeping our notations for the various characteristics separate, we facilitate the study of the correlations and of the still more instructive imperfections of correlation, and escape the danger of reading preconceived opinions into our system.

Princeton University Observatory
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November 21, 1934

SYSTEMATIC DISPLACEMENTS OF LINES IN THE SPECTRA OF CERTAIN BRIGHT STARS*

By WALTER S. ADAMS AND ELIZABETH MACCORMACK

ABSTRACT

Measurements of high-dispersion spectrograms taken with the coudé spectrograph of the 100-inch reflector show that the sodium lines D_1 and D_2 in the spectra of β Orionis, α Cygni, α Orionis, α Scorpii, α^1 Herculis, β Pegasi, and ϵ Pegasi give systematically larger negative radial velocities than the normal stellar lines. The D lines in the spectra of several of the stars are unsymmetrical and evidently consist of two components, one of which is a normal stellar line. The measured radial velocities are in full accord with this conclusion.

Results agreeing with those found for the D lines are given by the H and K lines, and, in the case of the M-type stars, by the pair of *Al* I lines between H and K. The average deviation of these lines, most of which are blends, from the normal stellar lines is of the order of -5 km/sec. Less accurate observations of several *Si* II lines in the spectrum of β Orionis indicate differences in the same direction but less in amount.

All these stars are relatively near, and it seems improbable that interstellar absorption can account for the observed results. For the M stars at least, the hypothesis of a gradually expanding envelope appears more satisfactory. Interstellar absorption may perhaps be more effective in the cases of β Orionis and α Cygni.

In the spectrum of γ Cygni the lines of neutral iron give larger negative velocities than those of ionized iron and titanium, and lines of *Ce* II still larger negative values. Similar but larger differences between neutral and enhanced lines are found in the spectra of α Canis Majoris and α Cygni. The hypothesis of radial convection currents affecting lines of different levels differently seems to be adequate to account for these results.

The absence of differential displacements of considerable size among absorption lines of different elements and of very different energy-levels is always a matter of surprise to one who investigates stellar spectra. Radial velocities derived from lines of various kinds, and based upon wave-lengths measured in the solar spectrum, are in almost all cases in as close agreement with one another as the accuracy of the measures will permit. This fact is the more remarkable in view of the great differences in size and density between giant stars and the sun. The natural conclusion is that radial convection currents, the most effective source for producing differences of displacement among spectral lines of different levels, do not as a rule very greatly exceed in velocity those believed to exist in the sun. There is, however, substantial evidence for the presence of stellar convection currents of quite appreciable importance.[1]

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[1] St. John and Adams, *Mt. W. Contr.*, No. 279; *Ap. J.*, **60**, 43, 1924.

Of the lines known to show marked differences of displacement, the detached or stationary lines, H and K of ionized calcium and D_1 and D_2 of neutral sodium, have long been recognized, and are very generally ascribed to the absorption of interstellar gases. At first observed only in O and B stars of early type, they have recently been found by Joy in stars of later type such as certain very distant variables.[2] A few unidentified lines discovered by Merrill in the spectra of some early-type stars may also be detached lines.[3] On the other hand, systematic differences in the displacements of certain lines, especially those of hydrogen and helium, found by many observers in the spectra of spectroscopic binaries, can in essentially all cases be ascribed to the combination of the spectra of the two components.

The large difference between the displacements of the bright hydrogen lines and those of the ordinary absorption lines in the spectra of long-period variables of type Me, and in many stars of types S, R, and N, is of a different character from those already mentioned, and almost certainly originates in the atmosphere or the immediate neighborhood of the star itself. The presence of strong convection currents affecting hydrogen emission lines differently from the absorption lines is probably the most satisfactory hypothesis so far advanced to account for the observations.

Differences in the displacements of stellar absorption lines, except in the case of certain peculiar spectra usually of early type, are normally very small and must be studied with the aid of a large amount of observational material or with high dispersion. Small differences between the radial velocities given by lines of ionized and neutral elements, as well as between those of lines of different elements, were noted by Adams[4] many years ago in the course of a study of large-scale spectrograms of a few bright stars. R. H. Curtiss and other members of the staff of the Detroit Observatory in numerous publications discussed important differences found among different lines, particularly in the spectra of Cepheid variables; and these stars were also studied by Jacobsen[5] and others with varying results. Some

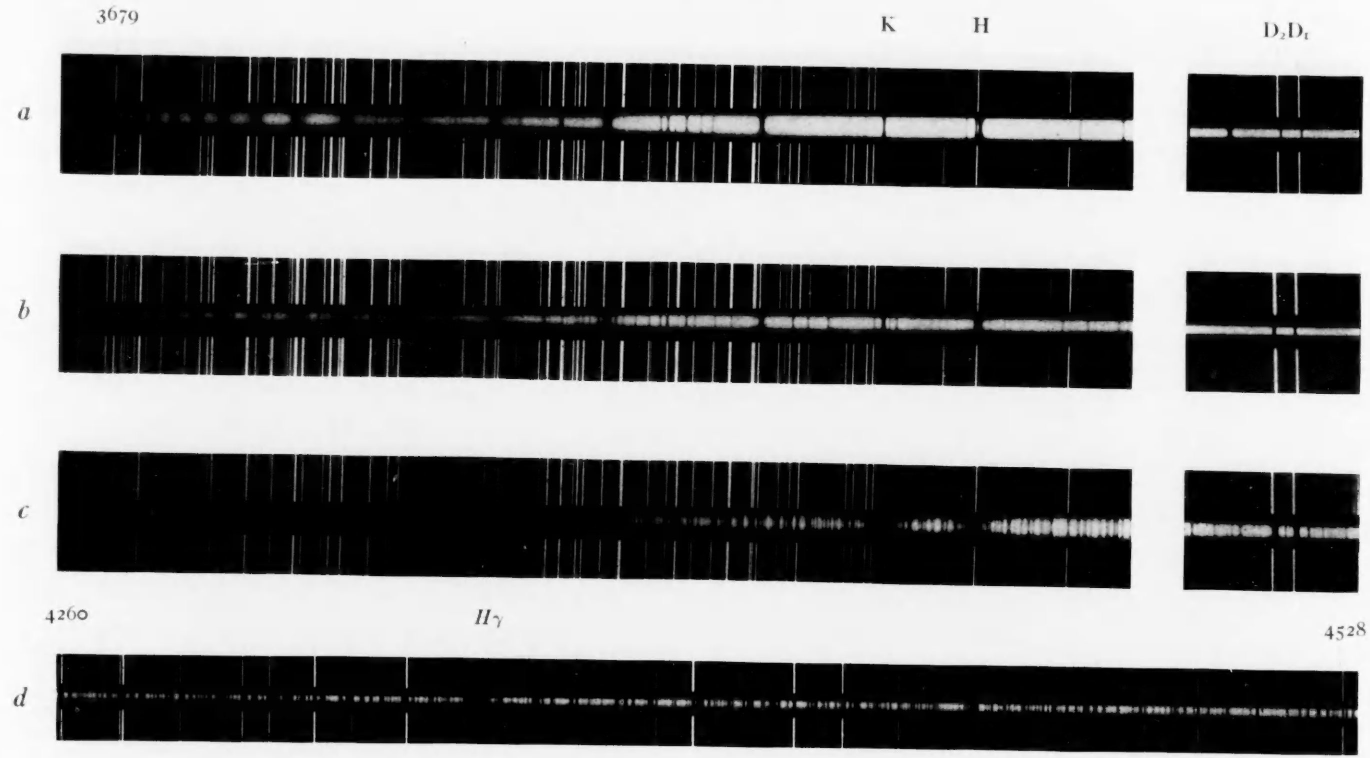
[2] *Pub. A.S.P.*, **46**, 51, 1934.

[3] *Ibid.*, 206.

[4] *Mt. W. Contr.*, No. 50; *Ap. J.*, **33**, 64, 1911.

[5] *Lick Obs. Bull.*, **12**, 138, 1926.

PLATE IV



TYPICAL COUDÉ SPECTRA

a) β Orionis　　b) α Cygni
c) α Orionis　　d) γ Cygni

years ago Adams and Joy[6] called attention to the systematic departure of the results for lines of one or two elements, especially *Ce* II, from those for other lines in the spectrum of γ Cygni.

The results given in this communication deal with the displacements of the lines of *Na* I, *Ca* II, *Al* I, *Si* II, and one or two other elements in the spectra of a few bright stars photographed with high dispersion at the coudé focus of the 100-inch telescope. In addition, the lines of *Ce* II in the spectrum of γ Cygni have been studied, and the arc and enhanced lines in the spectra of γ Cygni, α Canis Majoris, and α Cygni. Three spectrographs were used, all of the autocollimating type. The first (15 p) has a focal length of 15 feet (4.57 m), with a dense flint-glass prism having an angle of $52\overset{\circ}{.}5$. With a suitable inclination of plates of different emulsions within the plate-holder, the spectrum can be photographed in excellent definition from λ 4100 to λ 6700. The second spectrograph (9 g) has a focal length of 9 feet (2.74 m), with a 6-inch plane grating, ruled 700 lines to the millimeter, which is exceptionally bright in the yellow and red of the first order. The third spectrograph (UV) also has a focal length of 9 feet, but utilizes an ultra-violet glass prism with an angle of 64°. In general, the 15-foot spectrograph has been used for the blue region; the grating spectrograph for the green, yellow, and red; and the ultra-violet instrument for the region to the violet of λ 4100. The linear scales of the spectrograms are: 3.6 A per millimeter at $H\gamma$ for the 15-foot spectrograph; 5.6 A per millimeter for the 9-foot grating spectrograph; and 7.7 A per millimeter at H and K for the ultra-violet spectrograph.

Of the stars investigated, β Orionis, α Cygni, α Orionis, and α Scorpii are known to show variations in radial velocity, and γ Cygni has frequently been suspected of variation. All these stars, as well as α^1 Herculis, β Pegasi, and ϵ Pegasi, upon which a few observations have been made, are extremely bright intrinsically and may be classed as supergiants; α Canis Majoris and δ Ophiuchi are of more nearly normal brightness for their spectral types.

The observations extend over a considerable period of time during which numerous changes were made in the optical and mechanical parts of the spectrographs. Furthermore, some of the spectrograms

[6] *Proc. Nat. Acad.*, **13**, 393, 1927.

were taken when the temperature control was less accurately maintained than at present. For these and perhaps other reasons the absolute radial velocities are probably not so accurate as the probable errors might lead one to expect. The material, however, is of excellent quality for such differential measures as are here involved. The system of wave-lengths employed in the reductions is, for stellar lines, that of the *Revised Rowland Table*, and for the comparison lines of the iron arc that adopted by the International Astronomical Union. In the case of early-type spectra showing lines not present in the solar spectrum, the wave-lengths used are from laboratory sources and have been taken from Miss Moore's *Multiplet Table of Astrophysical Interest.*

THE SODIUM LINES D_1 AND D_2

The sodium lines D_1 and D_2 have been measured in the spectra of β Orionis, α Cygni, and the K-type star ϵ Pegasi, and in those of several stars of type M. Table I contains the results of the measures. The radial velocity given by the normal stellar lines, with its probable error, is in the fifth column; that from the sodium lines is in the sixth column. For β Orionis, the velocity from the *Si* II lines appears in the last column. The number of lines measured varies greatly. In some cases only a few in the vicinity of the D lines were measured, while in others the entire blue section of the spectrogram containing many lines has been included. The D lines in the spectra of α Scorpii, α^1 Herculis, β Pegasi, and ϵ Pegasi show marked asymmetry, and two measures are given, one on the maximum (max.) of each line and the other on the whole width (w.w.), the latter in the last column. The lack of symmetry seems clearly to be due to the superposition of a line of greater violet displacement upon a normal stellar line. There is some evidence of similar asymmetry in the spectra of α Cygni and α Orionis, but in these cases only the whole width of the lines could be measured.

Table II gives the velocities measured in the ultra-violet region for β Orionis, α Cygni, α Orionis, and β Pegasi. All the spectrograms except 698, which is a second-order plate with the grating spectrograph, were taken with the 9-foot ultra-violet prism spectrograph. H and K and a large variety of normal stellar lines were measured upon all the spectrograms, and especial attention was given to

Si II lines in β Orionis, and the *Al* I pair at λ 3944 and λ 3962 in α Orionis. Many lines of the Balmer series of hydrogen were measured in the spectrum of β Orionis. On the high-dispersion grating spectrogram of α Cygni the H and K lines appeared unsymmetrical, and both the maximum and the whole width were measured.

TABLE I

Plate No.	Spec.	No. Meas.	No. Stellar Lines	Stellar Velocity	D_1 and D_2	*Si* II
				β Orionis cB8		
				km/sec.	km/sec.	km/sec.
572	15 p	3	5	$+15.1\pm1.11$	$+7.7\pm0.45$	$+13.9\pm0.92$
578	9 g	3	2	24.6 ± 0.40	15.4 ± 1.29	18.1 ± 0.46
615	9 g	1	6	$+18.2\pm0.94$	$+13.9\pm1.06$	$+16.7\pm0.00$
				α Cygni cA2		
417	15 p	1	44	-0.2 ± 0.41	-5.9 ± 1.57	....
503	15 p	2	60	-4.1 ± 0.14	11.0 ± 0.00	....
550	15 p	1	46	-3.6 ± 0.36	11.6 ± 0.13	....
558	15 p	1	50	-1.1 ± 0.33	13.0 ± 0.42	....
570	15 p	1	36	$+3.0\pm0.53$	8.7 ± 0.64	....
573	9 g	1	11	-0.6 ± 0.59	14.1 ± 0.85	....
651	9 g	1	2	-1.2 ± 3.52	13.5 ± 0.01	....
655	9 g	1	26	-4.5 ± 0.36	11.6 ± 0.81	....
676	9 g	1	6	-2.9 ± 0.20	9.1 ± 0.09	....
678	9 g	1	13	-5.2 ± 0.60	-10.0 ± 0.09	....
				α Orionis cM2		
46	15 p	1	11	$+16.4\pm0.30$	$+17.9\pm0.15$	....
47	15 p	1	13	17.1 ± 0.25	14.7 ± 0.38	....
211	15 p	2	15	24.7 ± 0.21	24.1 ± 0.13	....
219	15 p	1	13	20.6 ± 0.28	17.7 ± 0.47	....
231	15 p	1	12	23.7 ± 0.27	18.1 ± 0.39	....
449	15 p	1	22	18.2 ± 0.35	15.8 ± 0.16	....
571	15 p	2	22	22.6 ± 0.21	15.2 ± 0.18	....
590	9 g	1	21	20.9 ± 0.28	14.9 ± 0.89	....
611	9 g	2	40	17.9 ± 0.16	12.8 ± 0.17	....
616	9 g	1	23	18.3 ± 0.23	13.3 ± 0.42	....
617	9 g	1	25	17.5 ± 0.25	13.8 ± 0.38	....
621	9 g	1	31	16.0 ± 0.24	14.4 ± 0.42	....
625	9 g	1	17	$+16.5\pm0.18$	$+14.2\pm0.72$	....

TABLE I—*Continued*

Plate No.	Spec.	No. Meas.	No. Stellar Lines	Stellar Velocity	D_1 and D_2 Max.	D_1 and D_2 W.W.
			α Scorpii cM2			
				km/sec.	km/sec.	km/sec.
628.........	9 g	1	27	− 4.7±0.14	−18.2±0.17	− 6.6±0.08
633.........	9 g	1	5	3.6±0.53	17.7±0.55	8.6±1.27
650.........	9 g	1	30	6.6±0.25	17.1±0.64	10.6±0.57
654.........	9 g	1	37	4.3±0.21	17.5±0.59	12.1±0.33
667.........	9 g	1	20	4.2±0.36	23.4±1.94	11.3±0.49
669.........	9 g	1	24	− 6.3±0.22	−25.8±0.84	−12.0±1.91
			α¹ Herculis M5			
619.........	9 g	2	19	−35.6±0.34	−48.2±0.26	−39.5±0.25
670.........	9 g	1	14	−37.1±0.57	−46.5±1.34	−40.2±0.91
			β Pegasi M2			
671.........	9 g	1	20	+ 7.2±0.18	− 2.6±0.87	+ 2.7±0.52
674.........	9 g	1	19	+ 7.7±0.21	− 1.3±0.18	+ 1.6±0.56
			ε Pegasi cK0			
679.........	9 g	1	45	+ 2.5±0.14	− 8.4±0.08	+ 1.7±0.15
681.........	9 g	1	29	+ 3.3±0.14	− 3.9±0.03	+ 3.6±0.28

NOTES TO TABLE I

β Orionis.—The sodium lines are narrow, rather faint, but of good quality for measurement. In spite of the few lines measured and the large probable error, the large deviation from the stellar velocity is clearly real, and is confirmed by the values for H and K given in Table II. The values given by the *Si* II lines differ less than those of the D lines from the stellar velocity, but the negative difference is also confirmed by the *Si* II lines in Table II.

α Cygni.—The sodium lines are strong and well measurable, but slightly unsymmetrical, especially on the grating spectrograms of larger scale. The difference between the velocities given by the sodium lines and the normal stellar lines is definite and large.

α Orionis, α Scorpii, α¹ Herculis, β Pegasi.—In these giant M-type stars the sodium lines evidently consist of two components which are superposed and produce unsymmetrical lines. In the last three stars both the maximum of the lines and the whole width are measured. Although only the whole width is measured in the case of α Orionis, the velocity given by the D lines fluctuates with the stellar velocity as if two components were present.

ε Pegasi.—The D lines show a lack of symmetry, although the velocity derived from measures of their whole width differs little from the stellar velocity.

A comparison of the differences in velocity shown in Tables I and II between the normal stellar lines on the one hand, and the D lines,

TABLE II

H AND K, *Si* II, AND *Al* I

Plate No.	Spec.	No. Meas.	No. *H* and *He* Lines	*H* and *He* Velocity	H and K	*Si* II
				β Orionis		
				km/sec.	km/sec.	km/sec.
716	UV	2	16	+21.8±0.51	+15.8±0.51	+18.0±0.75
758	UV	2	16	15.1±0.32	9.8±0.17	10.6±0.08
763	UV	2	12	29.6±0.60	20.2±0.85	21.2±1.54
776	UV	2	26	+14.6±0.47	+ 8.1±0.00	+10.5±0.46

α Cygni

Plate No.	Spec.	No. Meas.	No. Stellar Lines	Stellar Velocity	H and K Max.	H and K W.W.
				km/sec.	km/sec.	km/sec.
698	9 g	2	23	− 7.8±0.26	−16.2±0.98	− 8.8±1.35
747	UV	2	23	− 0.4±0.41	....	− 4.8±0.75

α Orionis

Plate No.	Spec.	No. Meas.	No. Stellar Lines	Stellar Velocity	H and K	*Al* I
				km/sec.	km/sec.	km/sec.
713	UV	3	16	+19.8±0.27	+13.9±1.20	+ 9.6±0.22
725	UV	1	15	24.5±0.64	16.7±2.60	10.1±1.57
748	UV	2	18	21.9±0.30	16.2±0.18	14.6±1.54
755	UV	2	18	21.7±0.39	16.9±0.50	18.2±0.15
764	UV	1	11	20.0±0.78	14.4±1.53	12.9±2.24
769	UV	1	10	23.1±0.68	17.6±0.76	15.8±1.68
779	UV	1	11	21.5±0.34	15.5±0.61	15.1±0.11
794	UV	1	13	15.0±0.40	9.5±2.09	10.7±0.68
796	UV	1	14	+20.6±0.52	+14.7±0.73	+18.0±0.65
				β Pegasi		
753	UV	1	17	+13.4±0.39	+ 5.2±1.37	....

H and K, and the lines of *Al* I and *Si* II on the other, is of value as indicating the amount of the systematic displacements for the lines of these various elements, and the nature of the agreement of the results in widely different parts of the spectrum. Since the radial velocities of β Orionis, α Cygni, and α Orionis are all known to vary, and since at least some of the lines in the spectra of α Cygni and α Orionis show a lack of symmetry probably dependent upon the

TABLE III

DIFFERENCES, VELOCITIES FROM LINES AS LISTED *minus* STELLAR VELOCITIES

Star	D_1 and D_2	H and K	*Al* I	*Si* II (Yellow)	*Si* II (Violet)
β Orionis	−7.0 (3)	−6.8(4)	....	−3.1(3)	−5.2(4)
α Cygni	8.8(10)	2.7(2)	....	....	....
α Orionis	3.4(13)	5.8(9)	−5.9(9)	....	....
β Pegasi	5.3 (2)	−8.2(1)	....	....	....
α Scorpii	5.2 (6)	....	....	....	....
α^1 Herculis	−3.5 (2)	....	....	....	....

phase of velocity-variation, close agreement cannot be expected among spectrograms taken at different phases. A general comparison, however, is of interest (Table III). The number of spectrograms measured is given in parentheses.

The principal conclusions from these results are: (1) The sodium lines in the spectra of all these stars show a systematic displacement with reference to the normal stellar lines, amounting to approximately 5 km/sec. in the case of the M-type stars, and to probably slightly more for β Orionis and α Cygni; (2) the results for H and K are in satisfactory agreement with those for the sodium lines; (3) lines of *Si* II, in both the violet and the yellow regions of the spectrum of β Orionis, although subject to much greater uncertainty, seem to show systematic displacements in the same direction as the D lines and H and K, but probably somewhat less in amount; (4) the two fundamental lines of neutral aluminum at λ 3944 and λ 3961 show displacements nearly identical with those of H and K; (5) the displacements of all these lines with reference to the normal stellar lines are without exception negative.

It is doubtful whether any of these stars can be at distances greater than 200 or 300 parsecs, and the M-type stars are probably con-

siderably nearer. The distance of α Scorpii, on the very probable assumption that it is a member of the Scorpio-Centaurus group, is about 110 parsecs. Distances of this order are much too small to make it probable that the observed effects, at least in the case of the M stars, can be ascribed to the absorption of interstellar gases distributed in any uniform manner throughout space. A comparison of the range in radial velocity shown by the normal stellar lines in α Orionis and α Scorpii with that shown by the D lines, H and K, and the *Al* I lines indicates that the intensity of the abnormal line entering into these blended lines must be of the same order as that of the stellar component. Lines of such intensity, if of interstellar origin, should certainly be present in the spectra of very distant O and B stars, but there is no evidence whatever that lines of *Al* I occur in such stars. The same is true of *Si* II in the spectra of stars too early in type to show normal stellar lines of this element.

A further objection to the view that these lines are of interstellar origin is the fact that the observed displacement is in every case negative. These stars are all giants and their peculiar motions are doubtless small on the average. Their distances are such that the component of motion due to galactic rotation is also small. A brief computation shows that on the assumption of distances corresponding to absolute magnitudes of -3 to -5 and no peculiar motion, the difference between the stellar and the interstellar lines should range from $+1.6$ km/sec. to -0.3 km/sec. Four of the differences for the six stars should be positive. The observations, however, give values of about -4 to -6 km/sec. for the blended lines, values which for the abnormal components themselves would be approximately doubled.

A possible explanation of the origin of the abnormal lines in these giant M-type stars may be an envelope of gas surrounding the stars and expanding with a moderate velocity. The existence of such envelopes is definitely established in the case of novae, and a similar assumption accounts in a satisfactory way for the spectra of Wolf-Rayet stars[7] and some of the phenomena of broad emission lines in stars of early type. The great size of the M stars, the variability in light of three of the four stars in which

[7] Beals, *Pub. Dom. Ap. Obs.*, **6**, 95, 1934.

these displacements have been measured, and the recent evidence afforded by the eclipsing variable, ζ Aurigae, a late-type giant, for the existence of an enormous envelope about the star[8] favor a hypothesis of this general character. The close physical analogy of the fundamental lines of *Al* I to those of *Na* I, and the agreement of the displacements of the lines of *Al* I, *Na* I, and *Ca* II make it certain that the lines of all three elements are absorbed by gases similarly distributed about the stars.

The behavior of the sodium and calcium lines in the spectra of β Orionis and α Cygni is also difficult of explanation on the basis of interstellar absorption unless the distances of these stars are greater than seem probable, or it is assumed that abnormally dense interstellar clouds of absorbing gas occur in the direction of the stars. The sodium lines in the spectrum of β Orionis are faint and narrow and have the appearance associated with interstellar lines, while those in α Cygni are undoubtedly complex and include a stellar component. Recent observations of the companion of β Orionis, which is generally thought to be physically connected with the brighter star, indicate that the calcium lines are considerably weaker in the spectrum of the companion than in that of β Orionis itself. The spectral types are closely the same in both cases. Whether or not this difference may be an absolute-magnitude effect, these observations indicate that general interstellar absorption can be responsible in but small part for the intensities of the sodium and calcium lines in the spectrum of β Orionis. It seems reasonable to conclude that the hypothesis of an expanding envelope around both β Orionis and α Cygni may account for the observed results, although the effect of interstellar absorption is by no means ruled out entirely. The behavior of the *Si* II lines in the spectrum of β Orionis, which appear to show displacements in the same direction but somewhat less in amount than the sodium and calcium lines, is difficult to explain on either hypothesis. The observations of these lines are necessarily of low weight as compared with those of the sodium and calcium lines and the true amount of their displacement is still in doubt.

The spectrum of γ Cygni is of exceptional interest. It is almost identical with that of several Cepheid variables, the lines of the rare

[8] Guthnick and Schneller, *Sitz. Preuss. Akad. Wissenschaften*, No. 10, 1932.

earths are unusually strong, and the star is so bright that the spectrum can be studied with very high dispersion. The absolute magnitude of the star is about −2.1. The lines both of the neutral and of the ionized elements are well defined and can be measured with high accuracy, and the spectrum is peculiarly well adapted for studies of small differential displacements among different classes of lines. The radial velocity of the star probably varies through a small range, but in this respect the observations are not conclusive.

The results of the measures of 13 spectrograms of γ Cygni, all taken with the 15-foot prismatic spectrograph with a linear scale at $H\gamma$ of 3.6 A per millimeter, are given in Table IV. In addition to our own, measures by Sanford, Mayall, and Miss Fretz are included. The lines have been divided into three groups: lines of neutral iron; lines of ionized iron and titanium; and lines of ionized cerium. Similar comparisons between lines of neutral and ionized elements are given for α Canis Majoris and α Cygni. In these two cases the lines have not been separated according to elements; but most of the neutral lines are due to iron, and most of the ionized lines to iron, titanium, and chromium.

If the results in Table IV are collected and differences are taken, we find the following mean values:

	Ionized − Neutral	Ionized − *Ce* II
	km/sec.	km/sec.
α Cygni	+2.0	....
α Canis Majoris	1.2	....
γ Cygni	+0.71	+1.31

These results agree well with those found in the course of the earlier work at Mount Wilson, as regards both the sign of the displacements and their magnitude. Previous measures had given +0.90 km/sec. for the difference between ionized and neutral lines in the spectrum of α Canis Majoris, +0.65 in the case of γ Cygni, and +1.45 for the difference between the lines of ionized iron and similar elements and those of ionized cerium in γ Cygni.

The hypothesis of radial convection currents in the stellar atmospheres, with an upward motion at low levels and a downward motion

at high levels, appears adequate to explain these results.[1] The problem of the levels of absorption lines in the atmosphere of the sun or a star is a most complicated one, but it seems clear from observa-

TABLE IV

ARC AND ENHANCED LINES AND *Ce* II

Plate No.	Spec.	No. Meas.	*Fe* I	*Fe* II, *Ti* II	*Ce* II
			γ Cygni cF7		
			km/sec.	km/sec.	km/sec.
18	15 p	2	$-4.90\pm0.18(20)$	$-3.98\pm0.22(20)$	$-4.88\pm0.22(12)$
27	15 p	2	$5.87\pm0.10(23)$	$5.18\pm0.13(22)$	$6.84\pm0.17(18)$
32	15 p	2	$5.64\pm0.11(28)$	$4.42\pm0.14(24)$	$5.52\pm0.27(9)$
36	15 p	2	$6.24\pm0.12(22)$	$5.37\pm0.17(14)$	$7.72\pm0.23(12)$
38	15 p	2	$9.12\pm0.12(25)$	$8.37\pm0.11(23)$	$10.58\pm0.25(14)$
52	15 p	2	$6.17\pm0.10(22)$	$5.66\pm0.11(26)$	$7.74\pm0.27(10)$
204	15 p	1	$8.11\pm0.17(53)$	$7.56\pm0.15(46)$	$8.64\pm0.37(11)$
207	15 p	1	....	....	$10.10\pm0.33(7)$
398	15 p	1	$8.47\pm0.34(14)$	....	$9.20\pm0.84(8)$
481	15 p	1	$6.61\pm0.46(15)$	$6.14\pm0.30(20)$	$6.75\pm0.52(7)$
516	15 p	2	$6.87\pm0.15(29)$	$6.12\pm0.10(32)$	$6.70\pm0.24(12)$
699	15 p	1	$7.56\pm0.07(45)$	$6.89\pm0.11(28)$	$7.89\pm0.26(14)$
721	15 p	3	$-7.04\pm0.06(44)$	$-6.58\pm0.08(27)$	$-7.47\pm0.09(25)$
			α Canis Majoris A2		
Plate No.	Spec.	No. Meas.	Neutral	Ionized	
			km/sec.	km/sec.	
54	15 p	1	$-10.2\pm0.24(25)$	$-9.0\pm0.18(50)$	....
640	15 p	1	$10.6\pm0.46(16)$	$9.5\pm0.21(28)$	....
804	15 p	1	$10.7\pm0.24(13)$	$9.2\pm0.20(18)$	....
807	15 p	2	$-10.1\pm0.12(36)$	$-9.2\pm0.10(44)$	....
			α Cygni cA2		
503	15 p	1	$-6.2\pm0.48(16)$	$-3.3\pm0.22(77)$	....
680	15 p	1	$9.8\pm0.34(19)$	$-7.9\pm0.19(81)$	....
698	15 p	2	$9.4\pm0.57(9)$	$-6.8\pm0.28(10)$	....
747	15 p	1	$-0.1\pm1.09(14)$	$+0.5\pm0.60(15)$	....

tions that the distribution of the atoms in a state to absorb radiation in the case of lines of the ionized elements differs considerably from that for neutral elements. The high-level enhanced lines, according-

ly, would on this hypothesis show a downward motion relative to neutral lines of lower effective level, in agreement with the observations. The enhanced lines in the spectra of all three of the stars listed in Table IV are on the average considerably stronger than the lines of elements in the neutral state, and a very fair degree of correlation is found between groups of lines of different intensities and their corresponding displacements. All the lines of cerium which have been recognized in the spectra of the sun and stars belong to *Ce* II, the element being very easily ionized. It is among the elements of high atomic weight, and observations of the spectrum of the center and limb of the sun indicate that its lines belong to a low level. Its lines in γ Cygni are weaker than the majority of the neutral lines measured, and the negative displacement which they show is probably to be ascribed to their low relative level.

One other result of interest seems to be indicated by measures of iron lines of different temperature classes in the spectrum of α Canis Majoris. In the yellow-green part of the spectrum are a considerable number of lines of class *e*, or class V, according to King's system of classification. The results of measures on two spectrograms of two groups of lines, one of classes *a–d* (I–IV on King's system) and the other of class *e*, are as follows:

	a–d	*e*
	km/sec.	km/sec.
Coudé 640	$-10.6\pm0.46(16)$	$-8.9\pm0.44(9)$
807	$-9.6\pm0.25(17)$	$-8.6\pm0.44(12)$

The lines in the comparison spectrum used for the reduction of the stellar lines are all of classes *a–d*, and the difference, 1.3 km/sec., in the radial velocities given by the two groups is probably real. If so, it would indicate a higher level for the high-excitation lines of class *e*, in agreement with the enhanced lines which they resemble so closely.

Carnegie Institution of Washington
Mount Wilson Observatory
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ON THE RADIAL-VELOCITY VARIATION OF THE CEPHEID VARIABLE FF AQUILAE*

By ROSCOE F. SANFORD

ABSTRACT

Light-variation.—FF Aquilae is a Cepheid with a period of $4^{d}5$ and a visual light-range of 0.44 mag., the interval from maximum to minimum being $1^{d}99$, or 0.445 *P*.

Short-period velocity variation.—Observations of 1932 and 1933 establish a velocity variation whose curve is a reflection of the light-curve and whose amplitude is about 14 km/sec.

Long-period velocity variation.—The mean residuals for the other observing seasons are systematic and seem to be consistent with a change of 15.5 km/sec. in the systemic velocity in a period of 4109^{d}. Owing to gaps in the observations, the evidence is not conclusive and a definitive answer must be left to the next few years' observations. A maximum departure from the 1932–1933 curve apparently should occur within the next year or two. The corresponding velocity of the system as a whole would be approximately -22 km/sec.

Companion star.—The G0 companion, at least 6.5 mag. fainter than FF Aquilae, must be a dwarf if the two stars form a physical system and if the absolute magnitude of FF Aquilae is to agree with that called for by the period-luminosity law.

The circumstances of the discovery of the variability of FF Aquilae are concisely stated by C. M. Huffer:

> The star HR 7165 was first used as a comparison star in a group for testing the spectroscopic binary HR 7267. The first observations were made on May 14, 1926, and June 6, 1927. After further observations, it became apparent by August 1, 1927, that HR 7165 was variable and that HR 7260 and 7267 were constant. All subsequent observations were made using the last two stars for comparison.[1]

Curiously enough, all three stars had shown velocity variations and appear in Table I of Moore's *Third Catalogue of Spectroscopic Binaries*.[2]

Huffer's observations extended from July, 1927, to October, 1929, and revealed the Cepheid type of light-variation having the elements

$$\begin{aligned}&\text{Max}=\text{J.D. } 2425065.32+4^{d}4714E \text{ (G.M.T.)}\\&\text{Max}-\text{Min}=1^{d}99\\&\text{Range}=0.443 \text{ mag.}\end{aligned}$$

HR 7165 was subsequently designated as FF Aquilae.[3]

* *Contributions from the Mount Wilson Observatory, Carnegie Institution of Washington*, No. 508.

[1] *Pub. Washburn Obs. U. Wisconsin*, **15**, 205, 1931. [2] *Lick Obs. Bull.*, **11**, 141, 1924.

[3] Boss 4817; HD 176155; BD+17°3799; mag. 5.4; F5; 1900, $\alpha\ 18^{h}53^{m}8$, $\delta\ +17°13'.6$. There is a 12-mag. comp. at 6″ distance in p.a. 132°.

J. H. Moore[4] had announced the variability of its radial velocity on the basis of a range of 11 km/sec. shown by the early Lick three-prism spectrograms. Comparable ranges in the early values determined at the Mount Wilson and the Dominion Astrophysical observatories corroborate this conclusion, although, by themselves, they would not establish the variability.

Observations recommenced at Mount Wilson in 1931 further confirmed the velocity variation and showed that the period is that of the light-variation and that the velocity minimum and light-maximum are approximately synchronous, furnishing added proof of the Cepheid character of the light-variation. Observations during 1932, however, pointed to a change, at least in the velocity of the system. It appeared that a few spectrograms with the higher three-prism dispersion used at the Lick Observatory would supply desirable, independent evidence on this point. Through the kindness of Dr. Moore, 13 well-distributed spectrograms were specially taken for use in the investigation, thus making a total of 17 from the Lick Observatory. Six velocities determined at the Dominion Astrophysical Observatory were kindly called to my attention by W. E. Harper. Later observations, in 1933 and 1934, increased the total of Mount Wilson velocities to 36. Table I lists the results for each observatory separately, C as usual indicating observations with the one-prism spectrograph and 18-inch camera at the 100-inch reflector, and γ those obtained with similar equipment at the 60-inch reflector. V denotes the ultra-violet three-prism spectrograph, which, for all cases in Table I, was equipped with a 10-inch camera. The mean dispersion of the Lick spectrograms is about 10 A/mm, or approximately three times that of the others. All the velocities depend upon micrometer measures of prominent arc and spark lines for stars of spectral class F, selected on the basis of velocity consistency.

An assembly of all the observations about a single epoch by means of the period of light-variation showed an excellent agreement between the Lick velocities for 1933 and those from Mount Wilson in 1932 and the fore part of 1933, but a systematic departure for the other groups. None of the data bearing on the performance of the Mount Wilson spectrographs in 1914 and 1931 gave any evidence of

[4] *Pub. A.S.P.*, **36**, 145, 1924.

TABLE I

RADIAL-VELOCITY OBSERVATIONS OF FF AQUILAE

Plate	Date	G.M.T.	Phase	Obsd. Vel.	Curve Vel.	O−C
	Mount Wilson Observatory					
				km/sec.	km/sec.	km/sec.
γ 3336......	1914 May 3	21^h50^m	3^d655	−20.6	− 7.6	−13.0
3362......	10	23 30	1.782	32.0	18.6	−13.4
3409......	June 3	22 33	3.385	17.7	7.2	−10.5
3535......	July 10	21 10	0.085	20.0	15.0	− 5.0
3616......	Sept. 1	17 31	3.748	22.9	7.5	−15.4
18419......	1931 Sept. 1	18 49	2.027	23.3	17.0	− 6.3
18423......	3	18 35	4.017	15.3	9.0	− 6.3
18450......	26	17 04	0.126	20.6	15.3	− 5.3
V 40......	27	16 04	1.085	26.0	21.5	− 4.5
γ 18462......	28	16 20	2.096	18.6	16.3	− 2.3
18868......	1932 May 25	20 07	0.797	20.8	20.9	+ 0.1
C 6061......	June 23	20 54	3.002	(2.5)*	9.1	....
γ 18925......	25	20 00	0.493	17.6	18.9	+ 2.3
V 203......	July 19	17 13	2.020	16.7	17.0	+ 0.3
γ 18970......	Aug. 10	17 19	1.667	22.9	19.5	− 3.4
18978......	11	16 24	2.628	12.4	12.0	− 0.4
18991......	13	17 52	0.219	18.8	16.4	− 2.4
19015......	16	19 58	3.306	7.6	7.5	− 0.1
19019......	17	18 19	4.237	8.2	11.1	+ 2.9
V 224......	18	16 19	0.683	20.1	20.3	+ 0.2
264......	Oct. 22	14 23	3.002	4.9	9.0	+ 4.1
270......	Nov. 16	14 23	1.174	20.0	21.5	+ 1.5
275......	17	14 39	2.181	15.0	15.6	+ 0.6
282......	18	14 43	3.188	8.4	8.0	− 0.4
γ 19515......	1933 Mar. 3	1 36	0.328	17.2	17.5	+ 0.3
19523......	5	0 47	2.294	(20.6)*	15.0	....
γ 19587......	Apr. 5	23 26	2.937	8.6	9.4	+ 0.8
19647......	May 6	22 54	2.616	14.5	12.0	− 2.5
19707......	June 30	19 36	3.821	10.0	7.9	− 2.1
V 420......	Sept. 9	14 48	3.078	14.4	8.5	− 5.9
456......	Nov. 6	14 35	2.941	19.1	9.5	− 9.1
469......	8	14 30	0.466	29.1	21.4	− 7.7
475......	27	15 14	1.612	24.8	19.9	− 4.9
γ 20276......	1934 Mar. 25	0 36	2.746	18.1	10.9	− 7.2
20347......	Apr. 24	0 32	1.443	27.0	20.7	− 6.5
20412......	May 24	23 16	1.091	33.9	21.5	−12.4
20489......	June 28	19 27	0.160	23.3	15.7	− 7.6
20494......	July 1	18 28	3.119	−13.7	− 8.3	− 5.4
	Lick Observatory					
	1918 Oct. 23	15 41	0.863	−23.7	−21.0	− 2.7
	1919 June 10	19 34	1.692	22.4	19.3	− 3.1
	1922 June 11	22 52	0.158	14.7	15.6	+ 0.9
	July 21	19 49	4.256	−12.1	−11.4	− 0.7

* Velocity from a plate of very poor quality.

TABLE I—*Continued*

Plate	Date	G.M.T.	Phase	Obsd. Vel.	Curve Vel.	O−C
	Lick Observatory—*Continued*					
				km/sec.	km/sec.	km/sec.
20282W....	1933 May 15	$22^h 22^m$	2^d648	−10.4	−11.6	+1.2
20302W....	23	21 21	1.660	17.1	19.5	+2.4
20319W....	28	21 48	2.205	15.4	15.5	+0.1
20403Q....	June 20	22 50	2.900	11.0	9.6	−1.4
20447W....	July 5	22 18	4.460	13.7	13.6	−0.1
20454N....	6	21 32	0.955	21.7	21.4	−0.3
20455W....	7	22 1	1.975	17.8	17.2	−0.6
20470M....	10	22 9	0.510	19.3	19.0	−0.3
20479W....	11	21 32	1.482	20.5	20.7	+0.2
20492X....	13	21 12	3.468	6.9	7.2	+0.3
20504W....	15	21 48	1.021	22.9	21.5	−1.4
20518M....	18	17 26	3.839	11.8	8.0	−3.8
20524Q....	20	17 24	1.366	−20.9	−21.0	+0.1
	Dominion Astrophysical Observatory					
	1922 June 14†	....	3.079	−16.9‡	−8.5	−8.4
	17†	....	1.618	24.8‡	19.9	−4.9
	Sept. 7†	....	2.991	20.1‡	9.1	−11.0
	1930 Oct. 27§	....	1.401	26.6	21.0	−5.6
	Nov. 1§	....	0.874	17.8	21.1	+3.3
	4§	....	3.883	−11.7	−8.1	−3.6

† *Pub. Dom. Ap. Obs.*, **2**, 199, 1924.

‡ A systematic correction of +2.2 has been applied.

§ *Pub. Dom. Ap. Obs.*, **6**, 189, 1934.

the necessity for a systematic correction to the velocities obtained during these years. Further, almost all the Mount Wilson measures of 1933 and 1934 are checked by standard velocity stars observed on the same nights. Finally, since the first three values from the Dominion Astrophysical Observatory include the systematic correction of +2.2 km/sec., noted by Harper,[5] we can only conclude that the persistent departures from the velocity variation of 1932 and 1933 must be real. The excellence of the spectrum does not admit accidental errors of the magnitude involved. It therefore became necessary to look for a variation, presumably of much longer period, upon which the velocity corresponding to the Cepheid change is superposed.

[5] *Pub. Dom. Ap. Obs.*, **2**, 199, 1924.

The Lick observations of 1933 and those from Mount Wilson for 1932 and the fore part of 1933 were accordingly represented by a velocity-curve based on the fourteen normal places listed in Table II, wherein each good Lick plate was given twice the weight of a

TABLE II

NORMAL PLACES FOR FF AQUILAE

No.	Phase	Vel.	O−C	Wt.	Plate Nos. from Table I
		km/sec.	km/sec.		
1	0.274	−18.0	−1.2	1.0	γ 18991, 19515
2	0.510	19.3	−0.3	1.0	Lick 20470M
3	0.658	19.5	+0.6	1.5	γ 18925, V 224, γ 18868
4	0.988	22.3	−0.9	2.0	Lick 20454N, 20504W
5	1.174	20.0	+1.5	0.5	V 270
6	1.503	19.5	+1.0	2.5	Lick 20524Q, 20479W, 20302W
7	1.956	18.2	−0.7	1.5	γ 18970, V 203, V 275
8	2.090	16.6	−0.2	1.5	Lick 20455W, 20319W
9	2.727	11.8	−0.8	1.5	γ 18978, 19647, 19587
10	2.774	10.7	−0.1	1.5	Lick 20282W, 20403Q
11	3.165	7.0	+1.1	1.5	V 264, V 282, γ 19015
12	3.468	6.9	+0.4.	1.0	Lick 20492X
13	4.029	9.1	+0.1	1.0	γ 19707, 19019
14	4.460	−13.7	−0.1	1.0	Lick 20447W

TABLE III

ELEMENTS OF FF AQUILAE FOR 1932–1933

Preliminary Elements	Corr.	Adopted Elements
P=period, 4^{d}4714		4^{d}4714
e=eccentricity, 0.20	−0.13	0.07
ω=angle of periastron, 97°	−12°86	84°14
K=amplitude of velocity variation, 7.60	−0.49	7.11 km/sec.
T=time of periastron passage, J.D. 2425064.736	−0.255	5064.481
γ=velocity of system, −14.11	−0.26	−14.37 km/sec.

Mount Wilson plate. The corresponding preliminary elements derived by the methods of H. N. Russell[6] and E. S. King[7] were corrected by a least-squares solution made in accordance with Schlesinger's[8] procedure (Table III). The least-squares solution reduces Σpv^2 by

[6] *Ap. J.*, **40**, 282, 1914.

[7] *Harvard Ann.*, **81**, 231, 1923.

[8] *Pub. Allegheny Obs.*, **1**, 33, 1910.

60 per cent, and none of the final residuals (fourth column, Table II) much exceeds 1 km/sec. The upper part of Figure 1 shows the representation of the normal places (circles) by the velocity-curve based on the adopted elements, while Huffer's light-curve appears in the lower part. FF Aquilae is evidently an excellent example of a Cepheid with a velocity-curve which is practically the mirror-image of its light-curve. The phases given in Tables I and II and those used in

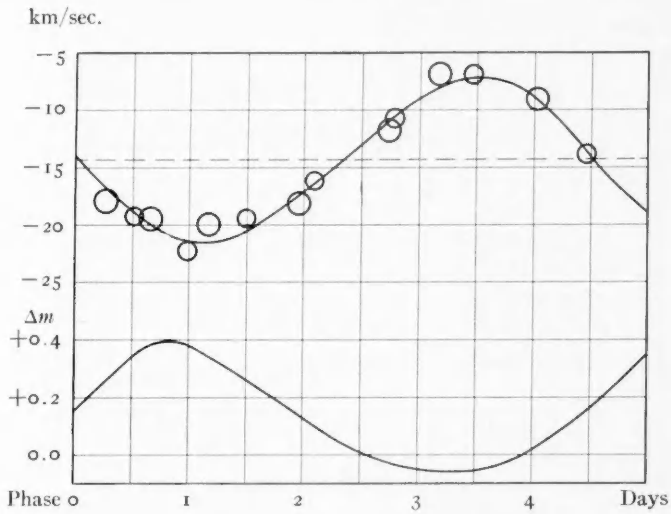

FIG. 1.—Above, radial velocity-curve for FF Aquilae. Smaller circles represent normal places from Lick observations; the larger, those from Mount Wilson observations (Table II). Broken line shows the velocity of the system. Below, Huffer's light-curve. Ordinates are differences from the comparison stars.

Figure 1 are reckoned from the adopted time of periastron passage, J.D. 2425064.481.

The velocities corresponding to the adopted elements, as scaled from the curve in Figure 1, are given in the sixth column of Table I. The residuals in the last column are satisfactory for all the observations used in deriving the elements; for the other measures they are, with few exceptions, negative. Unless the shape of the velocity-curve changes radically, the systemic velocity from the 1932–1933 measures is higher than for any of the other seasons.

The mean residuals by seasons, which appear in Table IV, are

assembled in Figure 2 with the aid of a period of 4109 days. Although there are gaps and the run of the residuals is none too smooth, the short-period velocity variation apparently oscillates about an axis which rises and falls with this period of 4109 days throughout a total range of about 15 km/sec.

The velocity of the system obtained for 1932–1933, −14.35 km/sec., is a maximum value. To obtain the velocity of the system as a

TABLE IV

MEAN SEASONAL RESIDUALS FOR FF AQUILAE

J.D.	O−C	J.D.	O−C
	km/sec.		km/sec.
2420315	−11.5	2426893	−0.5
2021	− 2.9	6984	+1.5
3255	− 4.9	7195	−0.4
6281	− 2.0	7375	−7.5
6603	− 2.0	7557	−7.8

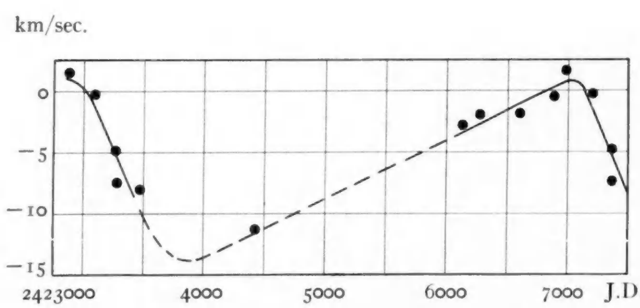

FIG. 2.—Curve of long-period change in velocity of the system of FF Aquilae. Points represent seasonal means of the residual velocities.

whole, this value should be decreased by approximately 7.5 km/sec., giving −22 km/sec. If Figure 2 is a correct picture of the variation in the center of mass velocity of the Cepheid star, a minimum is to be expected during 1935. The confirmation of this variation would establish the resemblance of the variable to the Cepheid system Polaris, in regard to a long-period variation in the velocity of the center of mass, as well as in period and spectrum.

It may be added that Huffer's observations are confined to an in-

terval during which not a single radial velocity was obtained and are centered about the mid-point of the rising branch of Figure 2. There is no particular reason to suspect spectral changes except during the short period of Cepheid variation, but on this point the evidence of our data is inconclusive. Observations covering the next seven or eight years will be needed to establish the result pictured (perhaps with overconfidence) in Figure 2 and to settle the questions of spectral changes during the longer period.

The spectrum of FF Aquilae (classified at Mount Wilson as cF6) includes many excellent lines, which are, however, less numerous than those in such stars as δ Cephei, and hence presumably less likely to be troubled by blends. This circumstance and the star's apparent brightness (mag. 5.0–5.4) render it particularly suitable for observation with high dispersion.

The companion star.—A single low-dispersion spectrogram of the twelfth-magnitude companion star in p.a. 132° is of course insufficient to determine whether the velocity of the companion agrees with that of FF Aquilae or not and is of little use in finding the absolute magnitude, but it does establish the type as G0. If the fainter star is a dwarf having the most frequent absolute magnitude for this type, the assumption that the two form a physical pair leads to an absolute magnitude for FF Aquilae that agrees with the period-luminosity law within the uncertainties in the assumptions; if a giant, the assumption of physical relationship will give for FF Aquilae an absolute magnitude that is almost certainly several magnitudes too bright. The value +0.3, given in *Mt. Wilson Contr.* No. 199, is in error, having been obtained on the same basis as for the general run of F stars before FF Aquilae was recognized as a Cepheid.

CARNEGIE INSTITUTION OF WASHINGTON
MOUNT WILSON OBSERVATORY
August 1934

ON THE RADIAL VELOCITY-CURVES FOR THE CEPHEID VARIABLE Y OPHIUCHI*

By ROSCOE F. SANFORD

ABSTRACT

Observations.—Forty-four radial velocities of Y Ophiuchi were obtained by Albrecht at the Lick Observatory in 1905–1906, and 39, mostly by the writer, at Mount Wilson in 1918–1934 inclusive. In addition he has measured 5 Lick spectrograms obtained by Jacobsen in 1928.

Curves of 1905–1906 and 1931–1932.—The radial velocity-curve from the 1931–1932 observations gives a larger eccentricity, amplitude, and angle of periastron than does the curve from the 1905–1906 observations. Both curves, with their steep branches from minimum to maximum velocity, are abnormal for Cepheids. The period of $17^{\rm d}1207$ does not bring the epochs of the two curves into coincidence.

Modification of the period from photometric data.—A least-squares solution for the elements of light-variation gave

$$\text{Maximum light} = \text{J.D. } 2408694.745 + 17^{\rm d}11934E \text{ G.M.T.}$$

Phases computed with this formula reconciled the two radial velocity-curves; the period and epoch were therefore adopted.

Radial velocities for the other seasons.—There is little choice between the representation of the 1918–1931 observations, since their phases are for the most part within the intervals for which the curves differ least. The 1933–1934 velocities are, however, definitely lower than either curve and show an unmistakable change, at least in the systemic velocity. The existing radial velocities therefore appear to establish changes in the shape of the radial velocity-curve and in the systemic velocity, but the seasonal groups are not numerous enough to determine the character of the changes.

Comparison of light- and velocity-curves.—Conclusions regarding the relation of light- to velocity-curves are perhaps legitimate if the curves are determined simultaneously or nearly so. Unfortunately this condition is not satisfied in this case. About all that can safely be inferred is that maximum light precedes minimum velocity and that minimum light and maximum velocity are roughly synchronous. In other words, neither velocity-curve is a good mirror-image of either light-curve.

More than a quarter-century has elapsed since radial velocity-curves were first obtained for Cepheid variables. That for Y Ophiuchi[1] depends upon studies by S. Albrecht[2] and others based upon 44 spectrograms which he obtained in 1905–1906. This paper derives a new curve from velocities obtained in 1931 and 1932 and compares it with the earlier one from Albrecht's observations, then considers the relation of the light- and velocity-curves after the mean period has been adjusted to give the best representation of all the data.

* *Contributions from the Mount Wilson Observatory, Carnegie Institution of Washington,* No. 509.

[1] HD 162714; BD$-9°4672$; α $17^{\rm h}47^{\rm m}3$; δ $-6°7'$ (1900).

[2] *Lick Obs. Bull.,* **4,** 132, 1907.

TABLE I

RADIAL VELOCITIES OF Y OPHIUCHI BETWEEN 1918 AND 1934*

Plate No.	Date	G.M.T.	Phase	Velocity	O−C
	Early Mount Wilson Observations				
			days	km/sec.	km/sec.
γ 7054	1918 June 22	18^h41^m	10.978	+ 0.5	− 0.5
9391	1920 July 27	17 40	6.564	− 1.2	+ 1.4
10257	1921 July 12	20 48	14.308	−12.8	− 6.0
13490	1925 June 13	18 30	8.069	+ 6.6	+ 2.7
	Recent Lick Observatory Observations				
75T	1928 April 4	23 17	7.227	+ 0.2	− 0.3
76O	9	23 39	12.242	+ 0.5	+ 2.0
78W	13	23 8	16.221	−10.0	+ 0.6
80O	29	21 56	15.043	− 8.8	− 0.6
81X	May 3	21 4	1.895	−13.1	+ 1.9
	Recent Mount Wilson Observations				
γ 18292	1931 June 5	20 7	17.098	−14.0	− 2.0
18297	27	20 53	4.892	− 8.4	+ 2.8
18303	28	20 36	5.880	−10.6	− 4.6
18310	29	21 27	6.916	+ 1.3	+ 2.1
C 5763	30	18 34	7.796	+ 3.2	− 1.0
γ 18323	July 2	16 52	9.725	+ 2.2	− 1.8
18338	22	20 26	12.754	− 1.5	+ 1.6
18341	23	19 19	13.708	− 3.0	+ 2.3
18345	24	20 5	14.740	− 9.7	− 2.0
18357	28	18 00	1.533	−17.6	− 3.0
18363	29	17 54	2.529	−12.6	+ 2.8
C 5824	Aug. 30	17 15	0.264	−12.3	+ 0.3
γ 18416	Sept. 1	15 25	2.187	−12.7	+ 2.3
18422	3	16 34	4.235	−14.5	− 1.0
18461	28	15 15	12.060	− 2.1	− 0.9
V 176†	1932 May 16	22 00	3.672	−20.5	− 6.0
C 6060	June 23	20 00	7.349	+ 4.9	+ 3.5
V 209	July 20	17 36	0.009	−13.6	− 1.6
γ 18977	Aug. 11	15 43	4.813	− 9.0	+ 2.5
18984	12	15 35	5.807	− 4.9	+ 1.6
19013	16	18 7	9.913	+ 6.6	+ 3.1
V 223	18	15 37	11.809	− 2.7	− 2.2
γ 19581	1933 April 5	0 28	1.506	−17.9	− 3.3
19588	6	0 23	2.502	−28.0	−12.7
19614	11	0 21	7.502	− 4.2	− 6.2
19646	May 6	21 53	16.280	−10.1	− 0.1

TABLE I—*Continued*

Plate No.	Date	G.M.T.	Phase	Velocity	O−C
	Recent Mount Wilson Observations—*Continued*				
			days	km/sec.	km/sec.
γ 19660	1933 May 15	20^h57^m	8.122	−11.8	−15.8
19679	June 2	21 38	9.030	+ 0.8	− 3.7
19706	30	18 51	2.675	−17.8	− 2.5
V 368	July 2	18 16	4.651	−17.1	− 4.6
375	4	16 23	6.573	− 8.9	− 6.4
γ 19750	13	16 9	15.563	−19.6	−11.7
19804	Aug. 12	16 44	11.349	+ 1.3	+ 0.8
20346	1934 April 23	23 28	8.840	− 1.0	− 5.5
20353	24	23 43	9.850	− 1.9	− 5.4

* The dispersion is approximately 35 A per millimeter at $H\gamma$ for all plates listed in this table.
† V series obtained with 3-prism ultra-violet spectrograph and 10-in. camera.

The new velocities include one in each of the years 1918, 1920, 1921, and 1925; 15 in 1931; 7 in 1932; 11 in 1933; and 2 in 1934—all obtained at the Mount Wilson Observatory. Five other velocities are available through the courtesy of Dr. J. H. Moore in permitting me to use the spectrograms obtained by T. S. Jacobsen at the Lick Observatory in 1928. Table I gives the data pertaining to the new material.

One-prism spectrographs with cameras of 18-inches focal length attached to the 100-inch (C series) and 60-inch (γ series) reflectors were used at Mount Wilson for all spectrograms except those of the V series. The velocities in the fifth column of Table I depend on the Mount Wilson tables for stars of class G.

At the close of the 1932 observing season a new velocity-curve was obtained from normal places based on the velocities for 1931 and 1932, with phases computed for each from the formula

$$\text{Maximum light} = \text{J.D. } 2425008.80 + 17^{d}.1207E \text{ (G.M.T.)} , \qquad (1)$$

made up of Ten Bruggencate's[3] epoch and Chandler's[4] period, which latter Albrecht had used. The slight change in the period finally adopted has no significant effect on the phases used for the new

[3] *Ann. Bosscha Sterrenwacht* (Lembang, Java), **2**, B53, 1927.

[4] *A.J.*, **24**, 5, 1904.

curve. The twelve normal places and observations included in each are listed in Table II.

The formal representation of the velocity-curve by the constants of orbital motion obtained from E. S. King's[5] standard curves was essentially correct, as may be seen from Table III giving the results of a least-squares solution; the corrections are in all cases less than the probable errors.

TABLE II

NORMAL PLACES FROM OBSERVATIONS OF Y OPHIUCHI IN 1931 AND 1932

No.	Phase	Velocity	O−C	Plates Involved
	days	km/sec.	km/sec.	
1	0.009	−13.6	−1.2	V 209
2	0.122	−13.2	−0.6	γ 18292, C 5824
3	2.083	−14.3	+0.8	γ 18357, 18363, 18416
4	4.242	−14.8	−1.3	18977, V 176
5	4.564	−11.4	+1.1	γ 18297, 18422
6	6.398	− 4.6	−1.7	18303, 18310
7	6.578	0.0	+2.0	C 6060, γ 18984
8	7.796	+ 3.2	−0.2	C 5763
9	9.725	+ 2.2	−1.5	γ 18323
10	10.861	+ 2.0	+0.5	19013, V 223
11	12.407	− 1.8	+0.3	γ 18338, 18461
12	14.224	− 6.4	+0.1	18345, 18341

Albrecht did not correct his elements by a least-squares solution, but this was done subsequently by both W. Zurhellen[6] and Miss Stella Udick.[7] The elements by Miss Udick, which give the smaller Σpv^2, are also listed in Table III. Her results for T, ω, K, and e are all smaller than the Mount Wilson values by amounts which exceed the sums of their respective probable errors. The difference of 1 km/sec. in the two values of γ is too small to be significant, especially as it has the same sign as the systematic difference between Lick and Mount Wilson observations given by Moore[8] for spectral class G. An interesting feature of both sets of elements is that ω is in the fourth quadrant instead of in the neighborhood of 90°, as for the typical Cepheid, which means that the steep branch of the curve is from minimum to maximum velocity, instead of from maximum to mini-

[5] *Harvard Ann.*, **81**, 231, 1923.

[6] *A.N.*, **177**, 329, 1908.

[7] *Pub. Allegheny Obs.*, **2**, 151, 1912.

[8] *Pub. Lick Obs.*, **16**, xxxi, 1928.

mum. Further, the greater values of ω and K show this peculiarity to be more pronounced in the later curve than in the earlier, a detail of importance in any attempt to superpose the two velocity-curves.

Up to this point $17^{d}1207$ had been used for the period; but a reduction of the curves to the same epoch with this value left a definite displacement which indicated the necessity of a correction. It seemed best to derive this from the observations of the light-maxima.

TABLE III

ELEMENTS OF Y OPHIUCHI

Element	Sanford (Mount Wilson)			Udick (Albrecht's Observations)
	Preliminary	Corr.	Final	
Period $=P$	$17^{d}1207$ (assumed)	....	$17^{d}11934$*	$17^{d}1207$ (assumed)
Periastron passage $=T$	J.D. 2425015.46	$+0^{d}352$	$15.812\pm0^{d}480$	$2408696.66\pm0^{d}756$ $2425012.69 (E=953)$†
Periastron angle $=\omega$	280°	$+6°85$	$286°85\pm12°00$	$201°66\pm16°7$
Eccentricity $=e$	0.250	-0.006	0.244 ± 0.034	0.163 ± 0.039
Semiamplitude of velocity variation $=K$	9.9	0.0	9.9 ± 0.5	$+7.70\pm0.30$ km/sec.
Systemic velocity $=\gamma$	-6.1	0.0	-6.1	-5.10 km/sec.

* Adopted from subsequent discussion.

† Udick's T brought forward to Sanford's with $P=17^{d}1207$.

Light-variation.—Table IV gives the observed epochs of well-determined maxima. The residuals in the third column, which correspond to the formula

$$\text{Maximum light} = \text{J.D. } 2408694.25 + 17^{d}1207E \text{ (G.M.T.)},$$

were used for a least-squares correction of the epoch and period, giving the new formula,

$$\text{Maximum light} = \text{J.D. } 2408694.745 + 17^{d}11934E \text{ (G.M.T.)}, \quad (2)$$

with residuals as in the last column of the table. The large residuals in the third column are all materially reduced without increasing

small values beyond the limits of error. Although the values tend to be negative in the first half of the table and positive in the latter half, these are rendered less noteworthy by several cases of opposite sign in each half. Ten Bruggencate's maximum is now represented within $0^d.7$. The lag of visual after photographic maximum to which

TABLE IV

LIGHT-MAXIMA FOR Y OPHIUCHI

Observer	J.D.(G.M.T.)	(O−C)₁	(O−C)₂
		days	days
Pickering*	2410012.88	+0.34	−0.05
Sawyer†	0902.82	+0.01	− .32
Sawyer	1587.86	+0.22	− .05
Yendell	1896.88	+1.07§	+ .82
Sawyer	1964.17	−0.12	− .37
Sawyer	2323.83	0.00	− .21
Sawyer	2683.11	−0.25	− .44
Sawyer	3042.92	+0.02	− .14
Luizet	4840.87	+0.30	+ .28
Yendell	5199.42	−0.69	− .67
Luizet	5936.52	+0.22	+ .28
Luizet	7015.14	+0.24	+ .40
Luizet	8145.09	+0.22	+ .47
Luizet	9240.82	+0.23	+ .56
Ten Bruggencate‡	2425008.80	−1.48	− .68
Bemporad‖	2419240.20	−0.40	−0.06

* *Harvard Ann.*, **46**, 160, 1903.

† This value and the twelve following are normal maxima given by Luizet in *B.A.*, **30**, 275, 1913.

‡ *Ann. Bosscha Sterrenwacht* (Lembang, Java), **2**, B53, 1927.

§ Luizet had $+0^d.07$ erroneously for $+1^d.07$.

‖ *Mem. Soc. Spett. It.* (2), **5**, 37, 1916. Bemporad's observations (1910–1912), found after the least-squares solution was performed, were combined by the writer into a mean light-curve from which the foregoing maximum was obtained. Although it has not contributed to the least-squares solution, its residuals furnish satisfactory confirmation of formula (2).

he has called attention is therefore cut in half and becomes less than some of the discrepancies among visual maxima themselves.

The two velocity-curves were then reduced to the same epoch with the new formula for maximum light. Miss Udick's curve was also lowered by 1 km/sec. in order to allow for the difference in the systemic velocities. The superposition (top of Fig. 1) thus found is as good as could be expected in view of the difference in the shape of the curves. The two crossing-points of Miss Udick's curve on the γ-axis fall outside of, but are symmetrical to, those of the other curve. For

both curves the phase for the time midway between minimum and maximum velocity is close to $5^{d}.9$.

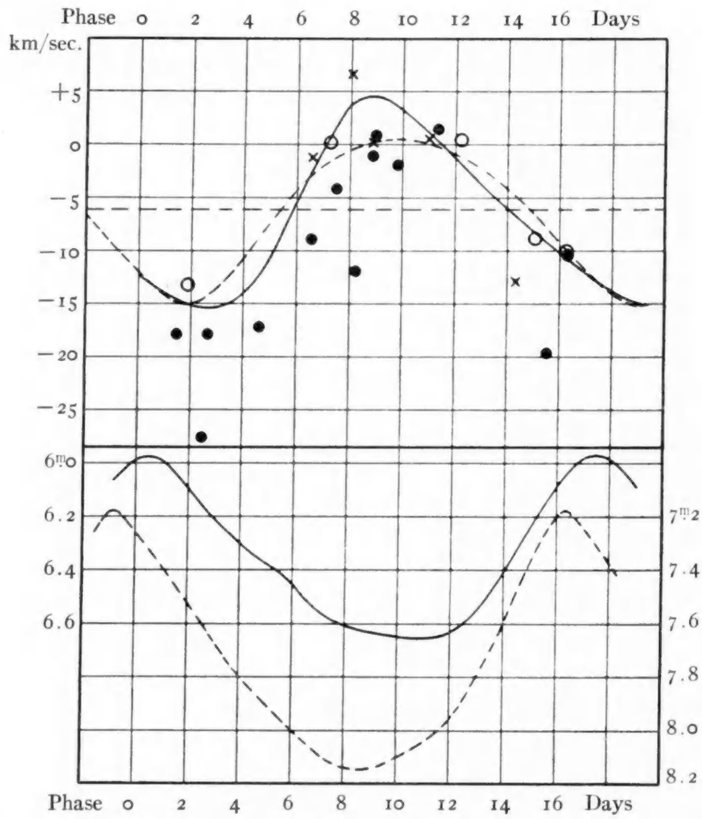

FIG. 1.—Above, velocity-curves of Y Ophiuchi; full line, that by Sanford from observations in 1931 and 1932 at Mount Wilson; broken line, Miss Udick's curve from Albrecht's observations at the Lick Observatory in 1905–1906. The 4 early Mount Wilson observations, the 5 from the Lick Observatory in 1928, and the 13 from Mount Wilson in 1933 and 1934 are represented by crosses, open circles, and solid circles, respectively.

Beneath, a visual light-curve by Luizet (upper) with ordinates to the left and a photographic light-curve by Ten Bruggencate with ordinates to the right.

The new formula, (2), therefore improves the representation of light-maxima and produces a satisfactory superposition of the two velocity-curves, ample justification for its adoption.

Other recent radial velocities.—The phases in both Tables I and II are derived from the modified expression (2). Figure 1 shows not only the superposed velocity-curves, but also, by crosses, the four 1918–1925 entries in Table I; by circles, the five of 1928 from the Lick Observatory; and, lastly, by dots, the 1933–1934 velocities obtained at Mount Wilson. The first two groups, except for the velocity of June 13, 1925, are so distributed in phase that they are about as well represented by one curve as by the other. The observations of 1933 and 1934, however, are outstanding, for, with one exception, they all fall below the 1931–1932 curve with a mean departure of -6 km/sec. During the 1933–1934 interval scarcely a plate of this variable was obtained which did not closely precede or follow one of a standard velocity star. Four measures of α Tauri, five of γ Aquilae, two of β Geminorum, and two of Arcturus check the performance of the spectrograph. In the mean these measures differ from the accepted velocities by only $+0.2$ km/sec. Hence these late velocities for Y Ophiuchi merit confidence. The simplest interpretation is a change of the order of -6 km/sec. in the value of γ, an assumption that would give much more reasonable, although far from satisfactory, residuals for the velocities of 1933 and 1934. At any rate, the observations for the last two seasons are consistent in showing a change in the nature of the velocity variation.

Comparison of light- and velocity-curves.—Conclusions in regard to the relation of light- and velocity-curves, unless observations have been made simultaneously, are probably always of doubtful certainty. In the case of Y Ophiuchi such conclusions are even more to be avoided, if we may trust the evidence given by Bemporad,[9] who shows that the curves determined by a number of observers, although spread over a comparatively few years, may differ radically. Ten Bruggencate's photographic light-curve (bottom curve, Fig. 1) from observations in 1927 stands closest in time to the 1931–1932 velocity-curve. Luizet's mean visual light-curve, above Ten Bruggencate's, is derived from observations which include the epoch covered by the earlier velocity-curve. In both cases the phases are referred to formula (2).

[9] *Mem. Soc. Spett. It.* (2), **5**, 37–43, 1916.

Neither is by any means simultaneous with either velocity-curve. As far as they go, the curves indicate that velocity minimum lags considerably behind light maximum, but not enough to identify the latter with the end of velocity of recession or minimum diameter on the pulsation hypothesis. Maximum velocity, on the other hand, and minimum light are roughly synchronous. Both light-curves show the steeper branch to be from minimum to maximum, in contrast with the velocity-curves, which have their steeper branches in the interval covered by decreasing light.

Carnegie Institution of Washington
Mount Wilson Observatory
August 1934

RADIAL VELOCITIES OF RR LYRAE IN 1928, 1929, AND 1930*

By ROSCOE F. SANFORD

ABSTRACT

Table I gives 20 unpublished radial velocities of RR Lyrae for 1928, 1929, and 1930. These values, together with those of two earlier lists, are all plotted in Fig. 1, with phases from Prager's formula for median light.

The radial velocities of the cluster-type variable RR Lyrae already published were obtained by Kiess in 1911 and 1912 at the Lick Observatory[1] and by various observers in 1916 to 1927, inclusive, at the Mount Wilson Observatory.[2]

Twenty additional spectrograms give values for the seasons of 1928, 1929, and 1930. These results appear in Table I.

The phases in the fourth column and those for the velocities of the other two lists were computed from Prager's[3] formula, which he has used so successfully in representing the existing photometric data, viz.:

$$\text{Median magnitude (G.M.A.T.)} = \text{J.D. } 2414856.4083 + 0^{d}56683735E$$
$$-0^{d}0693 \sin [0^{\circ}0155(E-1200)] + 0^{d}0086 \sin [0^{\circ}0544(E-325)].$$

Figure 1 shows all the radial velocities plotted with these phases, circles representing those from the Lick Observatory and dots those from the Mount Wilson Observatory. The curve of lesser amplitude is that obtaind by Kiess from the Lick Observatory measures; the other curve is by the writer from the first Mount Wilson series. Both have been adjusted to correspond roughly with the observations in the middle of their respective series. The recent Mount Wilson observations appear to be satisfied by the second curve as well

* *Contributions from the Mount Wilson Observatory, Carnegie Institution of Washington*, No. 510.

[1] *Lick Obs. Bull.*, **10**, 140, 1913.

[2] *Mt. W. Contr.*, No. 351; *Ap. J.*, **67**, 319, 1928; *ibid.*, **69**, 240, 1929.

[3] *Veröff. U.-Sternwarte Berlin-Babelsberg*, **5**, Heft 4, 1926.

as are those upon which the curve depends; hence no further improvement was attempted.

TABLE I

RADIAL VELOCITIES OF RR LYRAE FOR 1928, 1929, AND 1930

Plate	Date	G.M.T.	Phase	Velocity
			day	km/sec.
γ 15893	1928 June 20	20^h01^m	0.484	−53
15896	20	23 43	.072	101
15938	July 3	18 54	.401	56
16018	27	20 26	.091	107
16022	28	19 10	.471	47
C 4946	Aug. 27	15 28	.273	56
4953	28	15 33	.143	94
γ 16125	29	15 45	.018	98
16190	Sept. 27	18 27	.222	76
16203	29	16 29	.439	45
16246	Oct. 25	14 39	.288	94
16279	Nov. 5	15 10	.540	40
16719	1929 July 18	16 29	.516	48
16720	18	17 46	.002	62
16721	18	18 41	.041	103
16722	18	19 23	.070	105
16829	Aug. 20	15 46	.043	90
17490	1930 June 5	17 50	.040	89
17494	6	20 28	.016	75
17500	7	20 30	0.450	−44

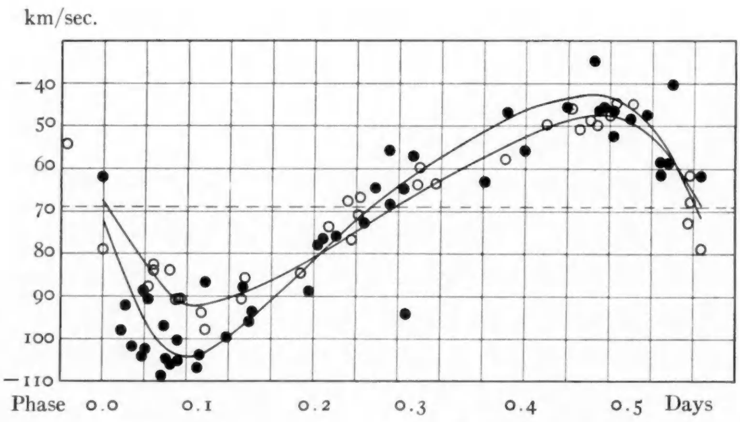

FIG. 1.—Velocity-curves of RR Lyrae. That of larger amplitude from Mount Wilson observations 1916–1927; the other, Kiess's curve from observations at the Lick Observatory in 1911 and 1912 (open circles). Dots are Mount Wilson velocities, including those added in 1928, 1929, and 1930.

The figure calls for no comment, beyond that already given, unless it be that a secondary minimum in the velocity variation between phases $0^{d}_{.}3$ and $0^{d}_{.}4$ is suggested by both the Lick and the Mount Wilson velocities. The six elements of orbital motion formally used to define the star's velocity variation must of necessity ignore this secondary effect. The observation of October 25, 1928, obtained with a dispersion of 75 A per millimeter at $H\gamma$ is inexplicably far off the curve.

No further observations at present are contemplated.

Carnegie Institution of Washington
Mount Wilson Observatory
October 1934

TRIGONOMETRIC PARALLAXES DETERMINED WITH THE 60- AND 100-INCH MOUNT WILSON REFLECTORS*

By ADRIAAN VAN MAANEN

ABSTRACT

A few stars of special interest, whose parallaxes are given in *Mt. Wilson Contr.* No. 506, are discussed in detail. Attention is also called to the considerable number of intrinsically faint stars that have been found during the last ten years. Well over 200 stars are now known whose photographic absolute magnitudes are 10 or fainter, and 12 which are even fainter than 15.

Mount Wilson Contribution No. 506 gives parallax results for 25 fields including 27 stars. Some of the stars require special comment.

An interesting star is No. 1166 in $h\chi$ Persei. In 1911 I found this star to have a proper motion of $0\rlap{.}''17$ annually; Oosterhoff called attention to its small color index; and Humason has recently found its spectrum to be A2. As the absolute parallax is $0\rlap{.}''014$, or, after applying the systematic correction, $+0\rlap{.}''007$, it is very likely that the star is a white dwarf with an absolute magnitude of about 8 or 9.

The parallax derived for the A-type star Lalande 5761, which on account of its large proper motion ($0\rlap{.}''900$) was supposed to be a white dwarf, is in such good agreement with results found at the Yerkes and McCormick observatories, as well as with the Mount Wilson spectroscopic parallax, that the mean value of $+0\rlap{.}''018 \pm 0\rlap{.}''002$ deserves great weight. The mean absolute magnitude is then 4.3, while its proper motion corresponds to a tangential velocity of 237 km/sec.

The three stars HD 45910, BD $+14^\circ 3887$, and HD 190073 were put on the program at the request of Merrill, as they all showed peculiar spectra. Since the mean of their parallaxes is negative, their luminosities must be rather high.

Wolf 860 was observed because, according to Baade,[1] it has a small color index. In measuring the plates the star was found to be a double, both components having the same proper motion of about

* *Contributions from the Mount Wilson Observatory, Carnegie Institution of Washington*, No. 507.

[1] *A.N.*, **239**, 359, 1930.

one-third of a second annually. Humason recently determined the spectra as M1 and M4, respectively. During our parallax determinations the following among the faint stars of large proper motion were also found to be double: Wolf 1328; Anon. $21^h24^m17^s$, $+11°43'$ (1900); and Ross 165. Ross 513, which has not yet been measured for parallax, also has a faint companion.

The fourteenth series of parallaxes brings the total of fields I have measured up to 350, including 390 objects. The material published in the first ten series was discussed in *Mt. Wilson Contr.*, No. 356. The great majority of the stars measured in the first 250 fields were selected to furnish material to Adams and his collaborators for their work on spectroscopic parallaxes. The last hundred fields, however, include, for the most part, faint stars of large proper motion in order to find very faint absolute magnitudes. This search has been very successful. While in 1924 we knew only very few parallaxes of stars with apparent magnitudes fainter than 10 (Schlesinger's *Catalogue*, published in that year, gives 47 such stars, of which about half were based on the older Groningen measures and have relatively little weight), I have since added 68 such faint stars, many of which have absolute photographic magnitudes fainter than 10.

In 1925 I published a list of stars whose photographic absolute magnitudes were 10 or fainter,[2] including 62 objects, of which 6 were extremely doubtful. Later Luyten published a list of some additional stars of faint absolute magnitude, raising the total to 86. At present we know 207 stars whose absolute photographic magnitudes are 10 or fainter. For 79 of these, trigonometric parallaxes have been determined at Mount Wilson.

The 207 objects are plotted in Figure 1, with spectral types as abscissae and photographic absolute magnitudes as ordinates. Stars whose spectra are at present unknown are plotted on the left side of the diagram. Mount Wilson determinations of trigonometric parallax are indicated by circles; physical companions of double stars, for which the parallax of the primary was used, are indicated by a cross. Two Ma stars are plotted on the Mo line with an arrow indicating that they will probably be moved to the right when better spectra become available. The diagram is probably fairly complete, al-

[2] *Pub. A.S.P.*, **37**, 24, 1925.

though possibly a few faint companions of double stars, whose magnitudes are in many cases rather uncertain, might be added.

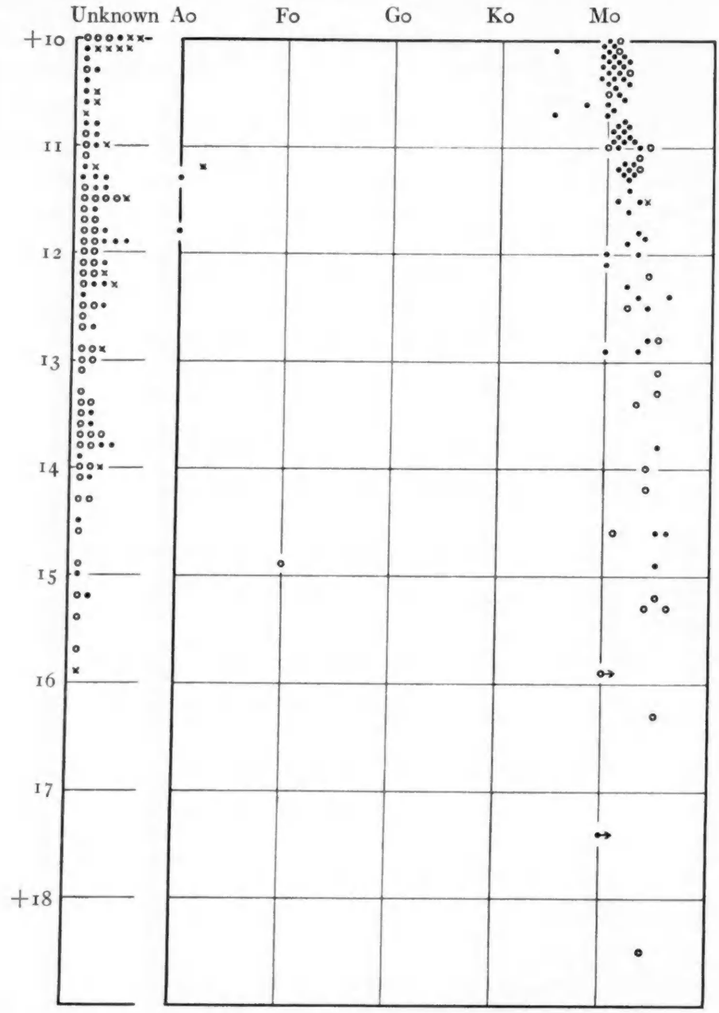

FIG. 1.—Distribution of stars with photographic absolute magnitude 10 or fainter according to spectral type (abscissae) and absolute magnitude (ordinates).

The diagram shows three very interesting points: (*a*) While for stars between 10 and 11 the spectral type is in the mean about M1, the fainter stars as a whole show a small but definite tendency to

have later spectral types. (*b*) Four white dwarfs are included, the companion of Sirius, the companion of o_2 Eridani, Greenwich A.C. $+70°8247$, and van Maanen's star. (*c*) Twelve stars have photographic absolute magnitudes fainter than 15. Schlesinger's *Catalogue*, published in 1924, included three such stars, and the companion of Procyon, although of uncertain magnitude, was also known at that time to be probably fainter than 15. In 1928 I was able to add one more, and during the last three years seven others. These twelve stars of extremely faint luminosity are collected in Table I.

TABLE I

STARS OF PHOTOGRAPHIC ABSOLUTE MAGNITUDE FAINTER THAN 15

Object	α 1900	δ 1900	β	Pg. *m*	Sp.	μ	π	Pg. *M*
Comp. Procyon	$7^h34^m\ 4^s$	$+\ 5°29'$	$+14°$	13.5?	.....	1.24	$+0''.303$	+15.9
Ross 619	8 6 32	+ 9 10	+23	14.2	M6	5.40	.163	15.3
Wolf 359	10 51 38	+ 7 36	+57	15.4	M4e	4.84	.413	18.5
Innes' star	11 12 0	−57 2	+ 3	12.5	.....	2.69	.340	15.2
Wolf 489	13 31 49	+ 4 13	+63	15.2	M	3.94	.140	15.9
Prox. Cent.	14 22 48	−62 15	− 2	13.0	M	3.85	.773	17.4
Barnard's star	17 52 54	+ 4 25	+12	11.5	M5	10.30	.541	15.2
Anonymous	18 57 41	−13 42	− 9	16.3	.....	0.80	.075	15.7
Comp. Ross 165	19 41 42	+26 55	0	15.0	.....	1.34	.119	15.4
Comp. Wolf 860	19 49 6	+18 30	− 6	17.7	M4	0.34	.034	15.3
Ross 248	23 36 58	+43 40	−17	13.8	M5	1.82	.319	16.3
No. 27, S.A. 115	23 37 46	+ 0 22	−58	16.5	.....	0.33	+0.056	+15.2

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LENS SYSTEMS FOR CORRECTING COMA OF MIRRORS

By FRANK E. ROSS

ABSTRACT

The only serious optical defect that is outstanding in the usual reflecting telescope system is coma. The various modifications which have been proposed, and in some cases introduced, to correct coma are described and their limitations are pointed out. On the initiative of Dr. G. E. Hale, the writer has investigated the optics of a simple lens system placed near the focus of a parabolic mirror, which will correct the outstanding coma and at the same time will not introduce other aberrations of a serious amount. Employing the third-order equations of general optical theory, a solution has been obtained for a two-piece lens system which eliminates coma and astigmatism but leaves outstanding spherical aberration and distortion. These, however, are not serious in the problems of photometry and astrometry requiring the use of a lens of this type. The third-order solution is found not to be unique, two parameters being undetermined. The double family of lenses is shown in Figure 2. It is not a simple matter to choose the one lens in this double family which will function best under any given set of conditions of focal length and aperture ratio. The difficulty is due to corrective terms introduced by the fifth- and higher-order aberrations. These can only be obtained by the ray-tracing methods customary in the computation of complicated lens systems. Corrective lenses have been computed by the writer for a number of telescopes, notably the Mount Wilson 100-inch and 60-inch, and have proved useful in problems of photometry and astrometry.

I. INTRODUCTION

The present investigation of the possibility of introducing a lens near the focus of a parabolic mirror to correct the outstanding error of coma was proposed to the writer by Dr. G. E. Hale, who had in mind its application to the proposed 200-inch telescope of the California Institute of Technology. Funds were made available for the computation of experimental lenses. The high speed-ratio, $F/3.33$, planned for this mirror makes a corrective lens of this type exceedingly desirable. As a basis of comparison, the performance of existing reflecting telescopes can be utilized in conjunction with the simple theory of coma, which gives

$$K''=\frac{3}{16}\left(\frac{D}{F}\right)^2\beta'' , \qquad (1)$$

where β'' is the angular distance of the star from the axis of the mirror, D the aperture, and F the focal length of the mirror. K'' is the total length of the comatic image in seconds of arc (distance AB

in Fig. 1). The breadth of the image is $\frac{2}{3}K''$. These values are only approximate.[1] Likewise Figure 1 is an approximation, the circles shown being asymptotic to series of double loops. The error becomes important only in the case of mirrors of very high speed-ratio and for large angular distances from the axis.

I am indebted to Dr. W. Baade for the numerical results in Table I, in which are given, in the case of four mirrors of well-known excellent performance, the distances from the axis at which the asymmetrical shading of coma first becomes visible. The theoretical length of the image (given by eq. [1]) corresponding to this observed angle and the observed diameter in arc of the minimum image are also given in Table I. Only plates taken on the best nights were examined. They are not of the finest grain, but are ordinary fast emulsions. An attempt will be made, from the data in Table I, to obtain some idea of the coma to be expected in the 200-inch mirror. The ratio of the theoretical length of the comatic image to the diameter of the axial image, L/M, will be utilized for this. It will be noted that this ratio is the same for the Lick and the Mount Wilson telescopes, but is materially less for the Yerkes and the Bergedorf instruments. The difference in L/M is undoubtedly due to differences in seeing and in focal length. Since the seeing conditions at the site of the 200-inch telescope will approximate those obtaining at Lick and at Mount Wilson, it is safe to assume that the ratio 1.35 will apply. Assuming further that the minimum image will be $1''.2$, we have $L = 1''.62$. Application of equation (1) then gives for the distance from the axis at which coma is first detectable, $\beta = 87''$, or 7 mm. The total field is then $2'.9$, or 14 mm. I am inclined, however, to increase this by 50 per cent, on account of the severity of the criterion for threshold coma adopted by Dr. Baade, as compared with other observers. The total field of good definition to be expected in the case of the 200-inch telescope is accordingly $4'.4$, or 21 mm on the plate, at the prime focus.

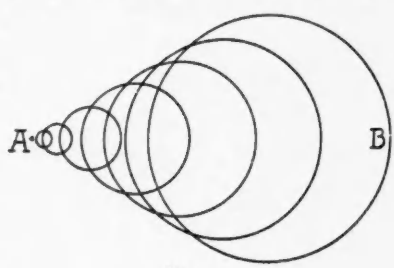

FIG. 1

[1] For a complete theory of coma see a paper by H. C. Plummer, *M.N.*, **62**, 352, 1901.

Pioneer work in improving the field of the reflecting telescope was undertaken by Schwarzschild.[2] Assuming a system consisting of a primary and a secondary mirror, he computed the form of each so that the outstanding error of coma is eliminated, at the same time preserving freedom from spherical aberration. The system which he finally adopted was of a Gregorian type, with the two concave mirrors separated by a distance of 1.25*F*, where *F* is the focal length of the system. The field is flat and the residual astigmatism is balanced on the two sides of the focal plane so that its effect is very small. Schwarzschild considers that the field of good definition is 2° in

TABLE I

Telescope	Aper. Ratio	Coma Vis. β	Theor. Length L	Min. Image M	L/M
Lick 36-in.	5.9	5'.0	1".61	1".2	1.34
Mt. Wilson 100-in.	5.0	3.6	1.62	1.2	1.35
Yerkes 24-in.	3.9	5.0	3.69	3.5	1.05
Bergedorf 1-meter	3.0	2.6	3.25	4.2	0.77

diameter. The diameter of the secondary mirror is one-half that of the primary. The focal plane lies between the two mirrors, at a distance from the secondary mirror equal to 0.4 the separation of the mirrors. The speed-ratio of the system is high, $F/3.3$, corresponding to that of the proposed 200-inch telescope. This is, moreover, secured by a very shallow principal mirror, the radius of curvature of which is 5.0*F*, a distinct advantage from an optician's standpoint. The radius of curvature of the secondary mirror is 1.67*F*. A disadvantage of the Schwarzschild type is the long tube-length. So far as I know, there is no example of the Schwarzschild system in operation. One is in course of construction by Professor W. A. Cogshall, of Indiana University. Its performance will be awaited with interest.

A modification of the Schwarzschild system for the elimination of coma has been made by H. Chrétien.[3] He substitutes a convex for the concave secondary mirror, as in the ordinary Cassegrain system, thus securing a telescope of short tube-length compared with its focal length. While in the Chrétien system spherical aberration, coma, and astigmatism are eliminated, the field curvature, given by

[2] *Göttingen Mitt.*, No. 10, 1905. [3] *Revue d'optique*, **1**, 49, 1922.

$R = 0.129F$, is relatively large, so that curved photographic plates must be used. The system has the further disadvantage, for many lines of work, of operating at a low speed ratio. The aperture of the small mirror is 0.3 that of the large one, and the separation of the mirrors is but $0.27F$, where F is the focal length. A telescope of this design, of 40-inch aperture and of about 270-inch focal length, has been constructed for the United States Naval Observatory by G. W. Ritchey. In this case the curvature of the field at 1° from the axis (4.7 in. on the plate) is 8 mm.

The problem of removing the outstanding error of field curvature of the Chrétien system was solved by M. H. Violette.[4] A lens system has been employed, consisting of a negative crown glass and a positive flint, placed in the plane of the main mirror, which is at a distance of one-quarter the diameter of the mirror in front of the focal plane.

Mention should be made of the system devised by R. A. Sampson[5] and applied to a Cassegrain type of speed-ratio $F/12.7$. In place of the usual secondary mirror, Sampson introduces a lens silvered on the back, which he calls a "reverser," of a diameter 0.4 that of the main mirror. The latter is not of parabolic form, but must be specially surfaced to correct spherical aberration. In addition, a pair of lenses of nearly equal and opposite powers and of the same glass, of a diameter equal to 0.3 that of the mirror, are placed in the axis at approximately two-thirds of the distance from the reverser to the focal plane. This system has not been constructed.

A very ingenious method of eliminating coma has been devised by B. Schmidt, of Bergedorf, Germany. In this system the main mirror is of strictly spherical form. At the center of curvature is placed a weak lens of optical glass, to correct the outstanding spherical aberration. Obviously, coma is corrected. The focal surface is, however, spherical, as in the Chrétien system, although of much less curvature. With this system remarkable photographs having a total field of 12° have been obtained by Mr. Schmidt.

II. ALGEBRAIC SOLUTION

The equations of third-order imagery should lead to an approximate solution, if any exists, of the problem of designing a lens of

[4] *Ibid.*, p. 397. [5] *Phil. Trans. R. Soc.*, **213**, 27, 1913–14.

zero power to correct the outstanding coma in a parabolic mirror. In this solution the thicknesses of the component lenses are neglected. From this point the solution must be carried further by the methods of ray-tracing, which take account of all higher-order terms and of the neglected thicknesses. The parameters involved in the solution are as follows: (1) focal length, F, or power, $\phi = 1/F$, of the mirror and of each lens of the correcting system; (2) indices of refraction μ and dispersion ν of the glasses used; (3) diameter of the mirror, D; (4) h-parameters, obtained by tracing a ray through the system by the equations of first-order imagery, the origin of the ray being the edge of the mirror and its direction being toward its focal point; the distance, d, from the axis at the point where this ray crosses each lens is proportional to the h for this lens, the relation being $d = (D/2)h$, so that h is a ratio; (5) g-parameters, obtained by tracing as in (4); in this case the origin of the ray is the vertex of the mirror and its direction is inclined 45° to the axis; the g's are the distances from the axis of the point where this ray crosses each lens.

The well-known general equations of third-order imagery are, in angular value:

$$\left.\begin{array}{ll}
\text{Axial color:} & C_a = D\sum \dfrac{h^2\phi}{\nu}\,, \\[2ex]
\text{Field color:} & C_f = -\tan\beta \sum \dfrac{gh\phi}{\nu}\,, \\[2ex]
\text{Spherical aberration:} & S_a = \dfrac{D^3}{32}\sum h^4\phi^3 A\,, \\[2ex]
\text{Coma:} & K = \dfrac{3D^2}{32}\tan\beta \sum (gh^3\phi^3 A - 2h^2\phi^2 C)\,, \\[2ex]
\text{Primary image surface:} & Q_p = P + 3\sum\left(\phi + g^2h^2\phi^3\dfrac{A}{4} - gh\phi^2 C\right), \\[2ex]
\text{Secondary image surface:} & Q_s = P + \sum\left(\phi + g^2h^2\phi^3\dfrac{A}{4} - gh\phi^2 C\right), \\[2ex]
\text{Distortion:} & J_a = \tfrac{1}{4}\tan^3\beta \sum\left(g^2\phi^2 T + g^3h\phi^3\dfrac{B}{2}\right), \\[2ex]
\text{Petzval sum:} & P = \sum \dfrac{\phi}{\mu}\,.
\end{array}\right\}\quad (2)$$

β is the field angle. The auxiliary quantities in equations (2) are

$$\left.\begin{aligned} A&=\frac{\mu+2}{\mu(\mu-1)^2}\,\sigma^2+\frac{4(\mu+1)}{\mu(\mu-1)}\,\sigma\pi+\frac{3\mu+2}{\mu}\,\pi^2+\frac{\mu^2}{(\mu-1)^2}\,,\\ B&=\frac{\mu+2}{\mu(\mu-1)^2}\,\sigma^2+\frac{4(\mu+1)}{\mu(\mu-1)}\,\sigma\epsilon+\frac{3\mu+2}{\mu}\,\epsilon^2+\frac{\mu^2}{(\mu-1)^2}\,,\\ C&=\frac{\mu+1}{\mu(\mu-1)}\,\sigma+\left(2+\frac{1}{\mu}\right)\pi\,,\\ T&=\frac{\mu+1}{\mu(\mu-1)}\,\sigma+\frac{1}{\mu}\,\epsilon\,, \end{aligned}\right\}\tag{3}$$

where σ is the shape factor of each lens, π the vergency factor, and ϵ the eccentricity factor. All of the factors have the same functional form, as follows:

$$\sigma=\frac{r'+r}{r'-r}\,,\qquad \pi=\frac{s'+s}{s'-s}\,,\qquad \epsilon=\frac{x'+x}{x'-x}\,,\tag{4}$$

where r and r' are the radii of the first and second surfaces, respectively, of the lens element; s and s' are object and image distances measured from the lens element; x and x' are similar to s and s', the object point being in this case the center of the entrance pupil, which is the vertex of the mirror. The radii are given by

$$r=\frac{2(\mu-1)}{\sigma+1}\,F\,,\qquad r'=\frac{2(\mu-1)}{\sigma-1}\,F\,.\tag{5}$$

The investigation will be confined for the present to a system of two lenses placed in the converging beam from the mirror. A system consisting of three lenses will be discussed in another paper. It may be found to have certain optical advantages, but the increased loss of light would be objectionable. In that which follows, the subscript 1 refers to the lens element nearer the mirror, the subscript 2 to the second lens element. A length is positive if generated by motion from its zero point, which is the lens element, to its other extremity, in the direction traveled by the light.

Let s_1 be the distance from the first lens element to the focus of

the mirror, a positive number. The remaining s's, determined by the equations of first-order imagery, are

$$\frac{1}{s_1'}=\phi_1+\frac{1}{s_1}\ ; \qquad s_2=s_1'-\Delta\ ; \qquad \frac{1}{s_2'}=\phi_2+\frac{1}{s_2}\ . \tag{6}$$

By definition $x_1=s_1-F_m$, a negative quantity. The x's are given by

$$\frac{1}{x_1'}=\phi_1+\frac{1}{x_1}\ ; \qquad x_2=x_1'-\Delta\ ; \qquad \frac{1}{x_2'}=\phi_2+\frac{1}{x_2}\ . \tag{7}$$

In these equations Δ is the separation of the lens elements. The parameters h and g are, then,

$$\left.\begin{aligned} h_1&=\frac{s_1}{F_m}\ ; \qquad h_2=h_1\frac{s_2}{s_1'}\ , \\ g_1&=s_1-F_m=x_1\ ; \qquad g_2=g_1\frac{x_2}{x_1'}\ . \end{aligned}\right\} \tag{8}$$

π and ϵ can now be obtained from equation (4). Putting for the present $\Delta=0$, $\nu_1=\nu_2$, and $F_2=-F_1$, we get

$$\left.\begin{aligned} \pi_1&=-1-2\frac{F_1}{s_1}\ , \\ \pi_2&=-\pi_1\ , \\ \epsilon_1&=-1-2\frac{F_1}{x_1}\ , \\ \epsilon_2&=-\epsilon_1\ . \end{aligned}\right\} \tag{9}$$

With these simplifications the third-order equations can now be rewritten:

$$\left.\begin{aligned} C_a&=0\ , \\ C_f&=0\ , \\ S_a&=\frac{D^3}{32}\,h_1^4\phi_1^3 w\ , \\ K_l&=\frac{3D^2}{32}\,\phi_1^2 h_1^2[w-2y]\tan\beta\ , \\ Q_p&=3\phi_1^2 h_1 g_1[\tfrac{1}{4}pw-y]\tan^2\beta\ , \\ Q_s&=\tfrac{1}{3}Q_p\ , \\ J&=\tfrac{1}{4}\phi_1^2 g_1^2[\tfrac{1}{2}pz+t]\tan^3\beta\ , \end{aligned}\right\} \tag{10}$$

in which

$$\left.\begin{aligned} w&=A_1-A_2\,,\\ y&=C_1+C_2\,,\\ z&=B_1-B_2\,,\\ t&=T_1+T_2\,,\\ p_1&=\phi_1 g_1 h_1\,. \end{aligned}\right\} \tag{11}$$

Inserting a change of variables,

$$\left.\begin{aligned} u&=\sigma_1-\sigma_2\,,\\ v&=\sigma_1+\sigma_2\,, \end{aligned}\right\} \tag{12}$$

and making use of equations (3), equations (11) become, for crown glass, $u=1.530$, and for flint, $u=1.630$:

$$\left.\begin{aligned} x&=v\begin{bmatrix}8.214u+12.480\pi_1\\ 5.611u+10.244\pi_1\end{bmatrix} \begin{matrix}\text{Crown}\\ \text{Flint}\end{matrix}\\ y&=\begin{matrix}3.120v\\ 2.561v\end{matrix} \qquad\qquad\qquad \begin{matrix}\text{Crown}\\ \text{Flint}\end{matrix}\\ z&=u\begin{bmatrix}8.214v+12.480\epsilon_1\\ 5.611v+10.244\epsilon_1\end{bmatrix} \begin{matrix}\text{Crown}\\ \text{Flint}\end{matrix}\\ t&=y \end{aligned}\right\} \tag{13}$$

Since we are concerned with improving the field of the mirror by means of a two-component lens system, without a material alteration of the focal length, attention will for the moment be given to K_l and Q_p in equations (10), which govern coma and astigmatism, respectively. The coma of the mirror itself is, by equation (1),

$$K_m=\tfrac{3}{16}D^2\phi_m^2 \tan\beta\,.$$

The astigmatism of the mirror is small and can be neglected. Putting $K_m+K_l=0$ leads to

$$\phi_1^2 h_1^2(pw-2y)+2\phi_m^2=0\,. \tag{14}$$

The condition that $Q_p=0$ gives

$$y=\tfrac{1}{4}pw\,, \tag{15}$$

so that

$$\left.\begin{aligned} w&=-\frac{4\phi_m^2}{p_1\phi_1^2h_1^2}\,, \\ y&=-\left(\frac{\phi_m}{\phi_1 h_1}\right)^2. \end{aligned}\right\} \tag{16}$$

Inserting in equations (16) the value of p (eqs. [11]) and replacing g_1 by its value s_1-F_m, or

$$g_1=(h_1-1)F_m\,, \tag{17}$$

we have finally

$$w=4\left(\frac{\phi_m}{\phi_1}\right)^3\cdot\frac{1}{h_1^3}\cdot\frac{1}{1-h_1}\,. \tag{18}$$

Spherical aberration can now be evaluated. It becomes, after simplification (eqs. [10]) and dropping subscripts 1:

$$S_a=\frac{1}{8}\left(\frac{D}{F_m}\right)^3\frac{h}{1-h}\,. \tag{19}$$

This is a useful and important formula, and shows at once the limitations of a two-piece correcting lens which must correct the coma of a mirror and at the same time be free from third-order astigmatism. It is seen that the only parameter at the disposal of the computer is h, the ratio of the distance from the correcting lens to the focus of the mirror. The powers and shapes of the lenses are immaterial.

Before making practical use of equation (19), it is necessary to know the relation between the spherical aberration given by this formula and the size of the confusion disk to which it gives rise. It is not difficult to show, either graphically or analytically, that the factor of dependence is approximately 0.20. The diameter of the confusion disk, which in our case becomes the diameter of the stellar disk in seconds of arc, is, then,

$$S_a=\frac{1}{40\,\sin\,1''}\left(\frac{D}{F_m}\right)^3\frac{h}{1-h}\,. \tag{19a}$$

Table II gives the diameter of the stellar disk due to outstanding spherical aberration, for all cases which might arise in practice, computed from equation (19*a*).

It has been found by computation that giving a small separation, Δ, to the components of the correcting lens will diminish the spherical aberration given by equation (19*a*). But it will be seen later that increasing Δ increases color aberrations, the admissible amount of

TABLE II

DIAMETER OF STELLAR DISK

h \ F/D	3.0	3.33	4.0	5.0	6.0
0.01......	1″.9	1″.4	0″.8	0″.4	0″.2
.02......	3.9	2.9	1.6	0.8	0.5
.03......	5.9	4.3	2.4	1.3	0.7
.04......	8.0	5.8	3.3	1.7	1.0
.05......	10.1	7.4	4.1	2.2	1.3
.06......	12.2	8.9	5.0	2.6	1.5
0.08......	16.6	12.1	6.8	3.6	2.1

which will determine its upper limit. Table II proves the desirability of keeping h small. Unfortunately, a small value of h is inconsistent with the equation for distortion, J. This is found to be, after laborious reductions, from the equations already given:

$$J=\frac{1}{4}\left(\frac{h-1}{h}\right)^2 \tan^3 \beta \,.$$

Distortion thus increases nearly as the inverse square of h and is proportional to the cube of the field angle, β. In addition to this difficulty in the matter of distortion, extensive numerical calculations show that when h is small the fifth- and higher-order terms in the astigmatism become important, which tend to reduce the angular extent of the field of good definition. It is thus seen that the computation of a correcting lens is by no means a simple, straightforward

matter. The effect of unavoidable outstanding aberrations must be weighed and judged with respect to the particular instrument for which the lens is to be designed, and with respect to the use to which it is to be put. The fields of greatest usefulness for such a lens are photometry and astrometry. In these fields, errors of spherical aberration in the correcting lens are, on account of the symmetry of the defect, of little importance. We are therefore justified in leaving a generous amount of this aberration uncorrected, with the result that distortion and errors of field are notably decreased.

III. COLOR CORRECTION

For a two-lens system the equations for the correction of color (eqs. [2]) are:

$$\left.\begin{aligned} h_1^2\frac{\phi_1}{\nu_1}+h_2^2\frac{\phi_2}{\nu_2}&=0\ ,\\ g_1h_1\frac{\phi_1}{\nu_1}+g_2h_2\frac{\phi_2}{\nu_2}&=0\ ,\end{aligned}\right\}\qquad(20)$$

which are obviously satisfied in the case of two thin lenses in contact, for then $h_1=h_2$, and $g_1=g_2$. The question arises whether it is possible to satisfy equations (20) for $\Delta\neq 0$. The necessary and sufficient condition for this is

$$\frac{g_2}{g_1}=\frac{h_2}{h_1}\ .\qquad(21)$$

It is easily shown that in general

$$\left.\begin{aligned} \frac{g_2}{g_1}&=1-\Delta\left(\phi_1+\frac{1}{x_1}\right)\ ,\\ \frac{h_2}{h_1}&=1-\Delta\left(\phi_1+\frac{1}{s_1}\right)\ .\end{aligned}\right\}\qquad(22)$$

Substituting in equation (21), we find $x_1=s_1$. Now x_1 is a large negative number and s_1 is a small positive number, subject to the restriction

$$s_1-x_1=F_m\ .$$

Accordingly, equation (21) cannot be satisfied, and it is therefore not possible to eliminate at once both of the color aberrations. This is true regardless of whether the resultant system is of zero power. It will be useful to obtain an expression for the outstanding C_f, assuming $C_a=0$. Equations (20) and (2) give in this case:

$$C_f=-h_1g_1\frac{\phi_1}{\nu_1}\left(1-\frac{h_1}{h_2}\frac{g_2}{g_1}\right)\tan\beta\,. \tag{23}$$

Put $\Delta(\phi_1+1/x_1)=G$ and $\Delta(\phi_1+1/s_1)=H$ in equations (22); then

$$\frac{h_2}{h_1}=1-H\,,\qquad \frac{g_2}{g_1}=1-G\,.$$

Noting that H and G are small fractions,

$$\frac{h_1}{h_2}\frac{g_2}{g_1}=1+H-G+\text{higher terms}\,.$$

Substituting in equation (23),

$$C_f=-h_1g_1\Delta\left(\frac{1}{g_1}-\frac{1}{s_1}\right)\frac{\phi_1}{\nu_1}\tan\beta\,.$$

Making use of the relations

$$h_1=s_1\phi_m\,,\qquad g_1=s_1-F_m\,,$$

we have, after simple reductions,

$$C_f=-\frac{1}{\nu}\frac{\Delta}{F_1}\tan\beta\,. \tag{24}$$

Useful deductions follow from equation (24). For a given Δ, it is seen that for a minimum C_f we must have F_1 and ν_1 large, which means component lenses of low power and low dispersion. Taking the extremities of the region of effectively operating wave-length at 4861 and 4047 A, which is approximately true for a star which is not too bright, we compute from Schott's catalogue of glasses a table

of ν (Table III). Since in an actual lens system Δ is never zero, and since, in order to reduce spherical aberration, it is desirable to increase Δ (p. 165), it is seen from Table III that flint glass must not be used. Assuming a K3 type of glass, Table IV has been computed, based on equation (24), which gives the length of the spectral image between the specified wave-lengths of a star, with arguments Δ/F_1

TABLE III

VALUES OF ν

Glass	Designation	ν
Fluor crown	FK2	74.2
Ordinary crown	K3	60.3
Light flint	LF3	40.7
Ordinary flint	F2	34.6

TABLE IV

LENGTH OF SPECTRAL IMAGE

Δ/F_1 \ β	10′	20′	30′
0.02	0″.2	0″.4	0″.6
.04	0.4	0.8	1.2
.06	0.6	1.2	1.8
.08	0.8	1.6	2.4
0.10	1.0	2.0	3.0

and β (β is the angular distance from the axis). This table will prove useful in designing correctors to fulfil certain requirements.

Returning to the first of equations (20), the condition for color correction on the axis, if the same glass is to be used for both components, is

$$\frac{\phi_2}{\phi_1} = -\left(\frac{h_1}{h_2}\right)^2 . \tag{25}$$

But if it is considered desirable to keep the powers the same, we have

$$\frac{\nu_2}{\nu_1} = \left(\frac{h_2}{h_1}\right)^2 . \tag{26}$$

The value of the right-hand member of these equations depends on Δ. In any actual lens system, owing to the thickness of the component lenses, its value in (26) cannot much exceed 0.93. Much time has been devoted to the relative merits of satisfying equation (25) or equation (26). The advantage of equation (26), which requires choosing different glass for the two components with dispersions differing by about 7 per cent, was so slight that it was decided in all cases to use the same glass for both components, and to correct axial color by equation (25), or, more strictly, by the accurate methods of ray-tracing. The power of the second component, in this method, was varied arbitrarily until the error of C_a was reduced to a value giving a stellar disk having a diameter less than 1″. Practically, C_f is obtained by computing the focal length of the entire system for the F (4861) and the h (4047) rays. The corresponding length of the spectral image of a star at any given distance from the axis is then easily computed. Equation (24) shows that this is directly proportional to Δ, for any given F. It has already been remarked that a large value of Δ improves spherical aberration, conflicting therefore with the requirement for a reduced spectral image. In general, the latter should be given preference, for, as already shown, a residual spherical aberration is not serious.

Nothing has been said so far about the focal length of the system or the change in focal point. On account of the necessary thickness of the lens elements it is not possible to design a system of strictly zero power and at the same time to satisfy the color equations. Separating the lens elements increases the disparity. However, the increase in focal length is seldom over 3 per cent, and the outward displacement of focus is not more than 3 inches, increases which in both cases are tolerable.

IV. COLLECTION OF FORMULAE

It will be useful to collect the formulae developed in the preceding pages for the computation of a zero-power correcting lens free from

coma and third-order astigmatism. It is assumed that the lenses are of zero thickness and that $\Delta=0$.

$$\left.\begin{aligned}
&s_1=\text{assumed distance of lenses from focus of mirror,}\\
&\phi_2=-\phi_1\ ,\\
&h_1=\phi_m s_1\ ,\\
&g_1=s_1-F_m\ ,\\
&\pi_1=-\left(1+\frac{2F_1}{s_1}\right),\\
&p_1=\phi_1 g_1 h_1\ ,\\
&w=4\left(\frac{F_1}{F_m}\right)^3\frac{1}{(1-h_1)h_1^3}\ ,\\
&u=1.520\left(\frac{1}{p_1}-\pi_1\right), \qquad \text{for } \mu=1.530\ ,\\
&v=0.0801 p_1 w\ , \qquad \text{for } \mu=1.530\ ,\\
&\sigma_1-\sigma_2=u\ ,\\
&\sigma_1+\sigma_2=v\ , \qquad a=1.060F_1 \qquad (\mu=1.530)\ ,\\
&r_1=\frac{a}{1+\sigma_1}\ , \qquad r_2=\frac{a}{\sigma_1-1}\ ,\\
&r_3=\frac{a}{1+\sigma_2}\ , \qquad r_4=\frac{a}{\sigma_2-1}\ .
\end{aligned}\right\} \tag{27}$$

These equations form the starting-point of the computation of correcting lenses. It is seen that there are two parameters of importance at our disposal, s and F, others being of minor importance. A double series of lenses has been computed from equation (27), with s and F as parameters, shown in Figure 2. Assuming F_m equal to 500 inches, s has been chosen equal to 10, 20, 30, and 50 inches, corresponding to values of h equal to 0.02, 0.04, 0.06, and 0.10. The value of F_1 ranges from $+50$ to -50 inches. The separations of the lenses shown in the figure are without significance. It is seen that in general the best forms result when the negative lens is in front, or nearer the mirror. For small values of h, which must be chosen when spherical aberration is to be especially well corrected, it is seen that the permissible range in F is small, and that F_1 must be negative. For larger values

of h it would be permissible to place the positive lens in front. Computation has shown, however, that in all cases a better lens results if the negative lens is in front; in particular there is better correction for spherical aberration and distortion.

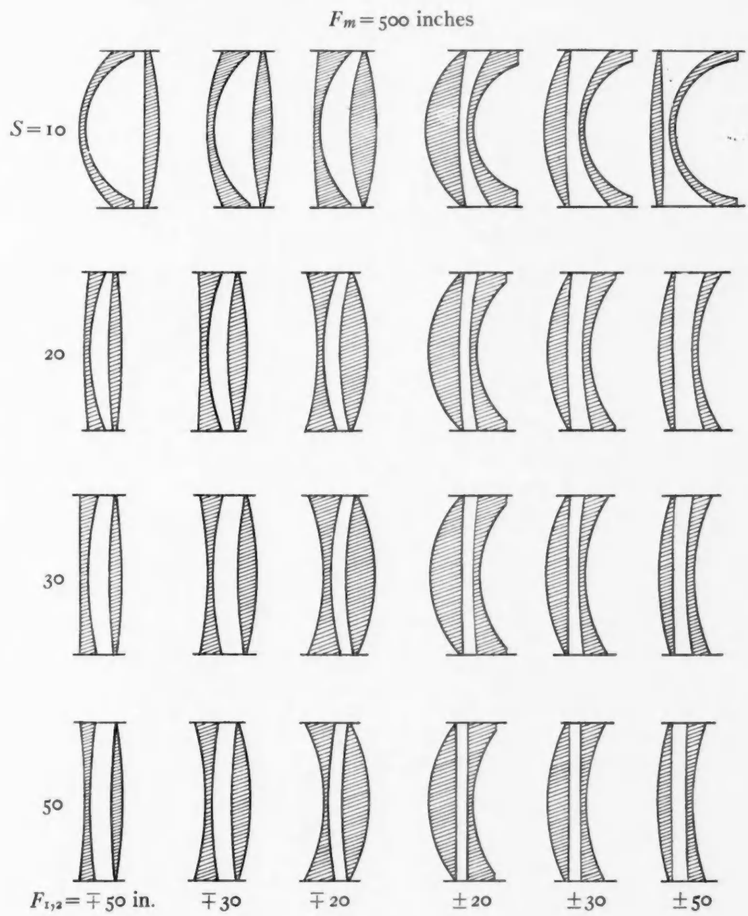

FIG. 2

V. COMPUTATION OF LENSES

The method of computation has been as follows: First, a value of h is chosen which will give the maximum admissible amount of spherical aberration. It is important that h be chosen not too small, be-

cause (*a*) the third-order distortion becomes unduly large; (*b*) the fifth- and higher-order terms in astigmatism become troublesome, unduly limiting the field of good definition. The next decision is a suitable value of the powers of the component lenses. If weak lenses are chosen, they become unduly meniscus-shaped (Fig. 2), and if strong lenses are chosen, the positive lens becomes unduly thick. There is between these extremes considerable latitude of choice. Third-order theory says nothing on this point except that which is contained in Table IV. The actual field of the correcting lens is determined by terms of the fifth and higher orders, which can be investigated only by numerical methods. Equations 27 serve only as the starting-point.

Correcting lenses, based on the theory of the present paper, have been designed by the writer for a number of observatories. The first of these was for use with the 60-inch telescope of the Mount Wilson Observatory and was constructed by the J. W. Fecker Company. It is of 8-inch aperture and is placed 15 inches within the focus of the mirror. Its performance has been described by the writer in the *Astrophysical Journal*.[6] Another one of similar design for use with the Mount Wilson 100-inch telescope was made by J. W. Dalton in the Mount Wilson optical shop. It is 12.5 inches in aperture and is placed 26 inches within the focal plane. Designs have been completed by the writer for correctors for the 200-inch telescope of the California Institute of Technology, and for the 80-inch telescope of the McDonald Observatory of the University of Texas.

YERKES OBSERVATORY
October 19, 1934

[6] **77**, 243, 1933.

NOTE

THE PHOTO-ELECTRIC COLOR OF β LYRAE

ABSTRACT

Photo-electric observations of the color of β Lyrae show it to be redder at primary minimum than at maximum. The amplitude of the variation is 0.030 mag.

The interpretation of the observations of β Lyrae constitutes one of the most interesting problems of astrophysics. In a recent article Struve[1] has pointed out several possible lines of attack. This note is the result of following up one of these, namely, the determination of the color of the star throughout its period.

The colors have been determined with the photo-electric photometer attached to the 12-inch telescope of the Yerkes Observatory. The photometer, which has been described by Stebbins,[2] contains a sensitized potassium photo-electric cell. A pair of filters are used which, when combined with the sensitivity of the cell, give effective wave-lengths of λ 4750 (for the yellow filter) and λ 4250 (for the blue filter). These filters give only 500 A between the effective wave-lengths, but experience has shown that colors can be determined with them as accurately as with a denser pair giving a larger spread between the effective wave-lengths. This is because the accuracy is limited by the quality of the sky and not by the instrument. Observations were made only on the most transparent nights.

Since the variation in brightness of β Lyrae is large, two comparison stars were chosen, their magnitudes being about the same as that of the variable at maximum and at minimum, respectively. These are γ Lyrae (mag. 3.3, spectral type A0p) and ζ Lyrae, a double (mags. 4.29 and 5.87, spectral types of both components A3). The star γ Lyrae has been found by Guthnick and Prager to be slightly

[1] *Observatory*, **57**, 265, 1934.

[2] *Ap. J.*, **74**, 289, 1931.

variable.[3] However, it was decided to use it as one of the comparison stars for the color determinations. The other comparison star is a double, and the components are measured together. This

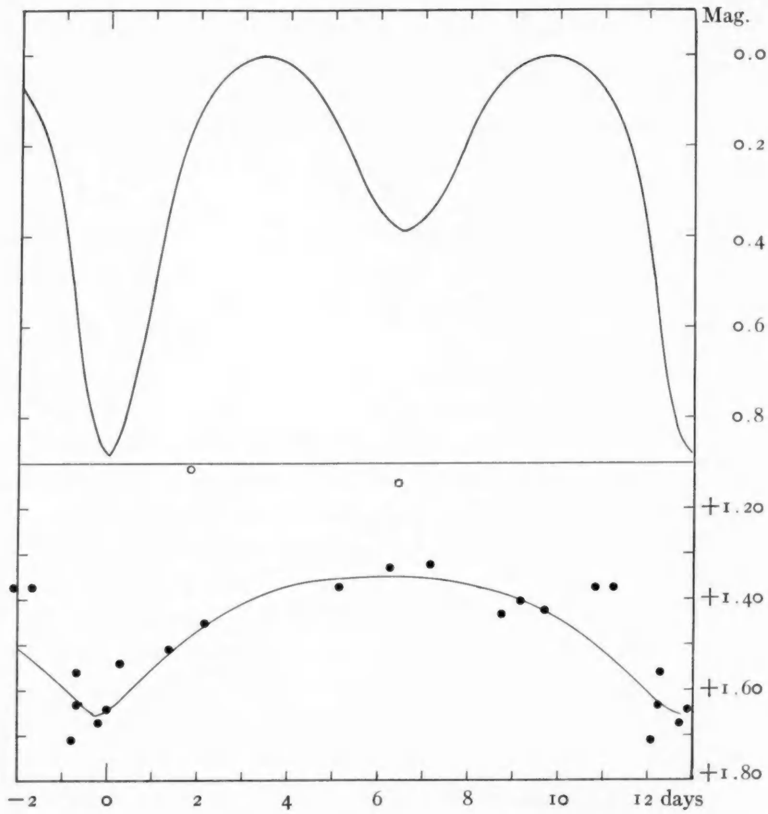

FIG. 1.—Observed variations in the color of β Lyrae (lower diagram). The ordinates are colors expressed in magnitudes and the abscissae are phases in days from primary minimum. A light-curve by Stebbins is shown in the upper part of the figure.

star has been found to be constant in light. Both comparison stars were used each night.

A double set of observations of color was made each night, each set consisting of a comparison with each star. One half the obser-

[3] *Kleinere Veröff. Sternwarte Berlin-Babelsberg*, **2**, No. 8, 1930.

vations of the variable came before and the other half after those of each comparison star. This served to check the variations of the transparency of the sky during the observations and tended to minimize any errors from this source. A constant extinction factor of 0.100 sec z was used throughout the series of observations and, since the differences in zenith distance between the variable and comparison stars are small, the variations of transparency from night to night were taken into account by assuming the color of the comparison stars to be constant and by correcting the colors of β Lyrae to the mean of the comparison stars.

The results are shown graphically in Figure 1. The ordinates are colors expressed in magnitudes and the abscissae are phases in days counted from the primary minimum. The observations were made during the months of March, April, May, and June. Only the very best nights were used, which accounts for the small number of observations at some phases. On two nights observations were made when the quality was questionable, and these are shown by circles in the diagram. The first of these is at phase 1.83 days, taken on March 16, 8^h51^m U.T. The variations in the readings taken at the telescope were found to be larger than expected, and an investigation showed that the sky was probably variable, a very slight haze having at times been noticed around Jupiter. The other observation is at phase 6.44 days, taken on June 19, 7^h51^m U.T., which was on the same night but 3.5 hours later than the observation at phase 6.29 days. The observing notes for the last part of that night indicated that the sky was becoming thicker.

The variation of the color of β Lyrae, as shown in the diagram, is small but seems to be quite definite. If a smooth curve is drawn through the observations, the maximum difference in color between the primary and the secondary minima is 0.030 mag., the star being redder at the primary minimum. The photo-electric photometer measures a faint star along with β Lyrae; Professor Rosenberg has obtained its color with his photographic photometer, and has found it to be decidedly bluer than the variable. It is obvious that the inclusion of this star would produce an effect opposite to that actually observed and shown in Figure 1.

Struve has used these observations of β Lyrae, in conjunction with his spectroscopic studies, to show that the secondary component of the eclipsing star is of later spectral type than the primary, and not of earlier type (B5) as has heretofore been supposed.

I am indebted to Mr. O. C. Collins for his assistance in reducing these observations.

C. T. ELVEY

YERKES OBSERVATORY
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